Such simulation studies are very useful. By repeating some experiments over and over again, we can generate measurements of discrete random variables that possess frequency distributions very similar to the probability distributions derived in this chapter, reinforcing the conviction that our models are quite accurate.

Exercises

3.1 When the health department tested private wells in a county for two impurities commonly found in drinking water, it found that 20% of the wells had neither impurity, 40% had impurity A, and 50% had impurity B. (Obviously, some had both impurities.) If a well is randomly chosen from those in the county, find the probability distribution for \( Y \), the number of impurities found in the well.

3.2 You and a friend play a game where you each toss a balanced coin. If the upper faces on the coins are both tails, you win $1; if the faces are both heads, you win $2; if the coins do not match (one shows a head, the other a tail), you lose $1 (win $(-1)$). Give the probability distribution for your winnings, \( Y \), on a single play of this game.

3.3 A group of four components is known to contain two defectives. An inspector tests the components one at a time until the two defectives are located. Once she locates the two defectives, she stops testing, but the second defective is tested to ensure accuracy. Let \( Y \) denote the number of the test on which the second defective is found. Find the probability distribution for \( Y \).

3.4 Consider a system of water flowing through valves from \( A \) to \( B \). (See the accompanying diagram.) Valves 1, 2, and 3 operate independently, and each correctly opens on signal with probability .8. Find the probability distribution for \( Y \), the number of open paths from \( A \) to \( B \) after the signal is given. (Note that \( Y \) can take on the values 0, 1, and 2.)

![Diagram](image)

3.5 A problem in a test given to small children asks them to match each of three pictures of animals to the word identifying that animal. If a child assigns the three words at random to the three pictures, find the probability distribution for \( Y \), the number of correct matches.

3.6 Five balls, numbered 1, 2, 3, 4, and 5, are placed in an urn. Two balls are randomly selected from the five, and their numbers noted. Find the probability distribution for the following:

a The largest of the two sampled numbers

b The sum of the two sampled numbers

3.7 Each of three balls are randomly placed into one of three bowls. Find the probability distribution for \( Y = \) the number of empty bowls.

3.8 A single cell can either die, with probability .1, or split into two cells, with probability .9, producing a new generation of cells. Each cell in the new generation dies or splits into two cells independently with the same probabilities as the initial cell. Find the probability distribution for the number of cells in the next generation.
3.9 In order to verify the accuracy of their financial accounts, companies use auditors on a regular basis to verify accounting entries. The company's employees make erroneous entries 5% of the time. Suppose that an auditor randomly checks three entries.

a Find the probability distribution for \( Y \), the number of errors detected by the auditor.

b Construct a probability histogram for \( p(y) \).

c Find the probability that the auditor will detect more than one error.

3.10 A rental agency, which leases heavy equipment by the day, has found that one expensive piece of equipment is leased, on the average, only one day in five. If rental on one day is independent of rental on any other day, find the probability distribution of \( Y \), the number of days between a pair of rentals.

3.11 Persons entering a blood bank are such that 1 in 3 have type \( O^+ \) blood and 1 in 15 have type \( O^- \) blood. Consider three randomly selected donors for the blood bank. Let \( X \) denote the number of donors with type \( O^+ \) blood and \( Y \) denote the number with type \( O^- \) blood. Find the probability distributions for \( X \) and \( Y \). Also find the probability distribution for \( X + Y \), the number of donors who have type \( O \) blood.

3.3 The Expected Value of a Random Variable or a Function of a Random Variable

We have observed that the probability distribution for a random variable is a theoretical model for the empirical distribution of data associated with a real population. If the model is an accurate representation of nature, the theoretical and empirical distributions are equivalent. Consequently, as in Chapter 1, we attempt to find the mean and the variance for a random variable and thereby to acquire numerical descriptive measures, parameters, for the probability distribution \( p(y) \) that are consistent with those discussed in Chapter 1.

**Definition 3.4** Let \( Y \) be a discrete random variable with the probability function \( p(y) \). Then the expected value of \( Y \), \( E(Y) \), is defined to be:

\[
E(Y) = \sum_y y p(y).
\]

If \( p(y) \) is an accurate characterization of the population frequency distribution, then \( E(Y) = \mu \), the population mean.

Definition 3.4 is completely consistent with the definition of the mean of a set of measurements that was given in Definition 1.1. For example, consider a discrete

2. To be precise, the expected value of a discrete random variable is said to exist if the sum, as given earlier, is absolutely convergent—that is, if

\[
\sum_y |y|p(y) < \infty.
\]

This absolute convergence will hold for all examples in this text and will not be mentioned each time an expected value is defined.