2.91 Can A and B be mutually exclusive if \( P(A) = .4 \) and \( P(B) = .7 \)? If \( P(A) = .4 \) and \( P(B) = .3 \)? Why?

2.92 A policy requiring all hospital employees to take lie detector tests may reduce losses due to theft, but some employees regard such tests as a violation of their rights. Past experience indicates that lie detectors have accuracy rates that vary from 92% to 99%. To gain some insight into the risks that employees face when taking a lie detector test, suppose that the probability is .05 that a lie detector concludes that a person is lying who, in fact, is telling the truth and suppose that any pair of tests are independent. What is the probability that a machine will conclude that

a each of three employees is lying when all are telling the truth?

b at least one of the three employees is lying when all are telling the truth?

2.93 In a game, a participant is given three attempts to hit a ball. On each try, she either scores a hit, \( H \), or a miss, \( M \). The game requires that the player must alternate which hand she uses in successive attempts. That is, if she makes her first attempt with her right hand, she must use her left hand for the second attempt and her right hand for the third. Her chance of scoring a hit with her right hand is .7 and with her left hand is .4. Assume that the results of successive attempts are independent and that she wins the game if she scores at least two hits in a row. If she makes her first attempt with her right hand, what is the probability that she wins the game?

2.94 A smoke detector system uses two devices, \( A \) and \( B \). If smoke is present, the probability that it will be detected by device \( A \) is .95; by device \( B \), .90; and by both devices, .88.

a If smoke is present, find the probability that the smoke will be detected by either device \( A \) or \( B \) or both devices.

b Find the probability that the smoke will be undetected.

2.95 Two events \( A \) and \( B \) are such that \( P(A) = .2 \), \( P(B) = .3 \), and \( P(A \cup B) = .4 \). Find the following:

a \( P(A \cap B) \)
b \( P(\overline{A} \cup \overline{B}) \)
c \( P(\overline{A} \cap \overline{B}) \)
d \( P(\overline{A} \cap B) \)

2.96 If \( A \) and \( B \) are independent events with \( P(A) = .5 \) and \( P(B) = .2 \), find the following:

a \( P(A \cup B) \)
b \( P(\overline{A} \cap \overline{B}) \)
c \( P(\overline{A} \cup \overline{B}) \)

2.97 Consider the following portion of an electric circuit with three relays. Current will flow from point \( a \) to point \( b \) if there is at least one closed path when the relays are activated. The relays may malfunction and not close when activated. Suppose that the relays act independently of one another and close properly when activated, with a probability of .9.

a What is the probability that current will flow when the relays are activated?

b Given that current flowed when the relays were activated, what is the probability that relay 1 functioned?

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selecting the correct answer is .25. If the student correctly answers a question, what is the probability that the student really knew the correct answer?

Two methods, $A$ and $B$, are available for teaching a certain industrial skill. The failure rate is 20\% for $A$ and 10\% for $B$. However, $B$ is more expensive and hence is used only 30\% of the time. ($A$ is used the other 70\%.) A worker was taught the skill by one of the methods but failed to learn it correctly. What is the probability that she was taught by method $A$?

Of the travelers arriving at a small airport, 60\% fly on major airlines, 30\% fly on privately owned planes, and the remainder fly on commercially owned planes not belonging to a major airline. Of those traveling on major airlines, 50\% are traveling for business reasons, whereas 60\% of those arriving on private planes and 90\% of those arriving on other commercially owned planes are traveling for business reasons. Suppose that we randomly select one person arriving at this airport. What is the probability that the person

a. is traveling on business?
b. is traveling for business on a privately owned plane?
c. arrived on a privately owned plane, given that the person is traveling for business reasons?
d. is traveling on business, given that the person is flying on a commercially owned plane?

A personnel director has two lists of applicants for jobs. List 1 contains the names of five women and two men, whereas list 2 contains the names of two women and six men. A name is randomly selected from list 1 and added to list 2. A name is then randomly selected from the augmented list 2. Given that the name selected is that of a man, what is the probability that a woman's name was originally selected from list 1?

Five identical bowls are labeled 1, 2, 3, 4, and 5. Bowl $i$ contains $i$ white and $5 - i$ black balls, with $i = 1, 2, \ldots, 5$. A bowl is randomly selected and two balls are randomly selected (without replacement) from the contents of the bowl.

a. What is the probability that both balls selected are white?
b. Given that both balls selected are white, what is the probability that bowl 3 was selected?

Following is a description of the game of *craps*. A player rolls two dice and computes the total of the spots showing. If the player’s first toss is a 7 or an 11, the player wins the game. If the first toss is a 2, 3, or 12, the player loses the game. If the player rolls anything else (4, 5, 6, 8, 9 or 10) on the first toss, that value becomes the player’s point. If the player does not win or lose on the first toss, he tosses the dice repeatedly until he obtains either his point or a 7. He wins if he tosses his point before tossing a 7 and loses if he tosses a 7 before his point. What is the probability that the player wins a game of craps? [Hint: Recall Exercise 2.119.]

**2.11 Numerical Events and Random Variables**

Events of major interest to the scientist, engineer, or businessperson are those identified by numbers, called *numerical events*. The research physician is interested in the event that ten of ten treated patients survive an illness; the businessperson is interested in the event that sales next year will reach $5$ million. Let $Y$ denote a variable to be measured in an experiment. Because the value of $Y$ will vary depending on the outcome of the experiment, it is called a *random variable*.

To each point in the sample space we will assign a real number denoting the value of the variable $Y$. The value assigned to $Y$ will vary from one sample point to another,
a If, as in Exercise 2.125, a test has sensitivity = specificity = .90, what is the positive predictive value of the test?

b Why is the value of the positive predictive value of the test so much higher than the value obtained in Exercise 2.125? [Hint: Compare the size of the numerator and the denominator used in the fraction that yields the value of the positive predictive value.]

c If the specificity of the test remains .90, can the sensitivity of the test be adjusted to obtain a positive predictive value above .87?

d If the sensitivity remains at .90, can the specificity be adjusted to obtain a positive predictive value above .95? How?

e The developers of a diagnostic test want the test to have a high positive predictive value. Based on your calculations in previous parts of this problem and in Exercise 2.126, is the value of the specificity more or less critical when developing a test for a rarer disease?

2.128 Use Theorem 2.8, the law of total probability, to prove the following:

a If \( P(A|B) = P(A|\overline{B}) \), then \( A \) and \( B \) are independent.

b If \( P(A|C) > P(B|C) \) and \( P(A|\overline{C}) > P(B|\overline{C}) \), then \( P(A) > P(B) \).

2.129 Males and females are observed to react differently to a given set of circumstances. It has been observed that 70% of the females react positively to these circumstances, whereas only 40% of males react positively. A group of 20 people, 15 female and 5 male, was subjected to these circumstances, and the subjects were asked to describe their reactions on a written questionnaire. A response picked at random from the 20 was negative. What is the probability that it was that of a male?

2.130 A study of Georgia residents suggests that those who worked in shipyards during World War II were subjected to a significantly higher risk of lung cancer (Wall Street Journal, September 21, 1978). It was found that approximately 22% of those persons who had lung cancer worked at some prior time in a shipyard. In contrast, only 14% of those who had no lung cancer worked at some prior time in a shipyard. Suppose that the proportion of all Georgians living during World War II who have or will have contracted lung cancer is .04%. Find the percentage of Georgians living during the same period who will contract (or have contracted) lung cancer, given that they have at some prior time worked in a shipyard.

2.131 The symmetric difference between two events \( A \) and \( B \) is the set of all sample points that are in exactly one of the sets and is often denoted \( A \triangle B \). Note that \( A \triangle B = (A \cap \overline{B}) \cup (\overline{A} \cap B) \). Prove that \( P(A \triangle B) = P(A) + P(B) - 2P(A \cap B) \).

2.132 A plane is missing and is presumed to have equal probability of going down in any of three regions. If a plane is actually down in region \( i \), let \( 1 - \alpha_i \) denote the probability that the plane will be found upon a search of the \( i \)th region, \( i = 1, 2, 3 \). What is the conditional probability that the plane is in

a region 1, given that the search of region 1 was unsuccessful?

b region 2, given that the search of region 1 was unsuccessful?

c region 3, given that the search of region 1 was unsuccessful?

2.133 A student answers a multiple-choice examination question that offers four possible answers. Suppose the probability that the student knows the answer to the question is .8 and the probability that the student will guess is .2. Assume that if the student guesses, the probability of