Statistical Inference 1650 Problem Session 7

1. Let $Y_1$ be the number of hours spent studying for a quiz. Let $Y_2$ be the grade (numerically represented). Suppose the joint distribution is given as below. What is the $\text{Cov}(Y_1, Y_2)$?

<table>
<thead>
<tr>
<th>$Y_2 \backslash Y_1$</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>2</td>
<td>1/9</td>
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<td>1</td>
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<td>0</td>
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2. Suppose we know that waiting times at a counter are well modeled by an exponential distribution but we don’t know the parameter ($\beta$). In order to estimate $\beta$ we take a random sample of size 3: $X_1, X_2, X_3$.

For each of the following estimators, determine its Bias and MSE.

(a) $X_1$
(b) $X_1 + X_2$
(c) $X_1 + X_2 + X_3$
(d) $X_1 + \frac{2X_2}{3}$

3. Suppose we have a circuit with voltage equal to $\mu$. We have a voltage meter but its readings are uniformly distributed between $\mu$ and $\mu + 1$. Since we know the readings are always greater than or equal to the true value, we decide to estimate $\mu$ by taking the minimum value of our readings.

(a) Suppose we take two readings with our meter $X_1$ and $X_2$. What is the density function of our estimator $\hat{\mu} = \min(X_1, X_2)$?
(b) What is the expected value $E[\hat{\mu}]$?

4. Using the normal distribution to approximate the binomial, resolve the following problem: A particular flight can only fit 200 people, but tickets were sold to 205 people. Suppose each ticket holder has a .05 chance of not showing up for the flight. What is the probability that the flight will be overbooked?