Statistical Inference 1650 Homework 4
Due Thursday October 9th

Homework is due by 1pm in the dropoff boxes located in the first floor of 182 George St. Show all of your work in deriving your solutions.

1. Suppose you are trying to login to a favorite website but you have forgotten your password. You know it is one of \( n \) passwords that you typically use. Lacking a better strategy, you start trying the possible passwords one at a time. Each time you try a password, you choose equally likely at random from all the passwords you haven’t tried before.

(a) What is the expected number of tries until you enter the correct password?

(b) What is the variance of the number of tries?

2. Using Markov’s inequality, bound the probability of:

(a) Flipping \( \leq 10 \) Heads from 100 flips of a fair coin.

(b) Flipping \( \leq 10 \) Heads from 100 flips of a biased coin with \( p(\text{Heads}) = \frac{1}{5} \).

Using Chebyshev’s inequality, bound the probability of:

(c) Flipping between 40 and 60 Heads from 100 flips of a fair coin.

(d) Flipping between 10 and 30 Heads from 100 flips of a biased coin with \( p(\text{Heads}) = \frac{1}{5} \).

3. Suppose your professor gives you 15 practice problems for the final exam. Instead of solving them all you decide to pick out 4 at random and do those. If you have trouble with more than 1 of them, you will take the time and do them all. Otherwise, you figure you are good for the final exam and stop studying.

(a) Suppose that of the 15 problems, you would have trouble with 5 of them. What is the probability that with your method you will do all the problems?

(b) When you initially pick out 4 of the problems, what is the expected number of trouble problems you will pick?

4. Let \( X \) be a random variable and \( \alpha \) and \( \beta \) be constants. Show that

\[
V(\beta X + \alpha) = \beta^2 V(X)
\]
(To think about: what is the variance of a constant function? Why does that make sense?)

5. Let $X$ be a random variable, define the random variable

$$Y = \frac{X - E[X]}{\sqrt{V(X)}}$$

What is the expectation and variance of $Y$? Give purely numerical answers.