1. A medical condition is carried by 1% of the population. A diagnostic test for the condition has the following accuracies. If an individual has the condition, the test correctly detects this 95% of the time. If an individual does not have the condition, the test incorrectly reads positive with probability 15%. Suppose person $X$ is given the test and the outcome is positive. What is the probability that person $X$ has the condition?

2. Consider a best-of-three tournament. There is a home team $A$ that has an advantage: team $A$ wins with probability $0.5 < p < 1$ when a game is played at home and $1 - p$ when played away. Suppose the tournament starts with a home game, then an away game, then a game at home (if necessary). Prove that Team $A$ would be more likely to win in a 1 game tournament than in the 3 game tournament.

3. Two people play a fun game! They take turns flipping a fair coin, the first to flip H wins.
   (a) What is the probability that the player who goes first wins?
   (b) What is the probability that the game lasts at least 4 rounds?
   (c) Given they have played 4 rounds, what is the probability that the game lasts at least 8 rounds?
   *(d) Suppose that the coin is biased with the probability of Heads equal to $p$. Suppose the game lasts exactly 4 rounds. What value of $p$ maximizes the chances of the game lasting exactly 4 rounds? For $k$ rounds?
   **(e) Suppose now that player 1 wins the game if the sequence HHT appears before the sequence HTT and player 2 wins if HTT appears before HHT. What is the probability that player 1 wins the game?

4. Suppose you forgot your password to your new phone. You know that it must be one of 10 that you always use. You decide to just start trying your passwords equally likely at random, never re-trying a password you’ve already tried. Unfortunately, after three failed attempts, your phone will lock and not let you try anymore. What is the probability that you try the correct password before your phone locks?

5. Suppose a drawer contains $x$ white socks and $y$ black socks. We are going to pack socks as follows: (1) Pick out a sock equally likely at random from all socks in the drawer and put it in your suitcase. (2) Pick out a sock equally likely at random from the drawer. If its color is the same as the last sock chosen then put in the suitcase and repeat from step (2). If its color is not the same, put it back in the drawer and repeat from step (1).

Find the probability that the last sock left in the drawer is white if:
(a) $x = y = 1$ (there are only two socks in the drawer to begin with)
(b) $x = 2$ and $y = 1$
*(c) For any value of $x$ and $y$