1. In a city, streets are laid out as a grid. Your home is at location (0, 0) and work is at location (10, 10). You walk to work taking only steps up and to the right.
(a) How many distinct ways can you walk to work?
Suppose you choose your path uniformly at random.
(b) If someone is standing at location (10, 0), what is the probability you will pass them? At location (9, 1)?

2. Suppose your materials science roommate managed to make a two-headed coin. Your roommate has the two-headed coin and a regular two-sided fair coin in their pocket. They draw a coin from their pocket equally likely at random and flip the coin. Given that the coin comes up heads, what is the probability that the fair coin was flipped?

3. Let $A, B, C$ be events of a common probability space.
(a) Prove the following equation holds:
$$p(A \cup B \mid C) = p(A \mid C) + p(B \mid C) - p(A \cap B \mid C)$$
(b) Give an example that shows the following does not generally hold:
$$p(A \mid B \cup C) = p(A \mid B) + p(A \mid C) - p(A \mid B \cap C)$$

4. Suppose a bucket contains one white and one black ball. We pick out a ball equally likely at random and then put the ball back along with another of the same color. Then we repeat. What is the probability that the first time we pick a white ball is after the $i$th iteration?

5. Suppose we have two identical biased coins: coin1 and coin2.
For both coins, the probability of Heads is $p$ and the probability of Tails is $q = 1 - p$.
Suppose we flip both coins. If the coins match, namely if we get two Heads or two Tails, we record nothing and flip both coins again. If the coins do not match, we stop and record the outcome of coin2. What is the probability of recording Heads?