Due before class on Friday, April 15th. It can be dropped off in the APMA 1650 homework box on the first floor of the APMA department, 182 George St OR at class (before it starts) on Friday.

APMA 1650: Complete all unstarred problems.
APMA 1655: Complete all unstarred and single starred problems.
Double starred problems are particularly challenging and I do not necessarily expect you to answer them to completion. You should work on them and write up your (partial) solutions/attempts/computations, etc.

Show all work!

1. You want to estimate \( \pi \) by throwing darts onto a wall 2 feet by 2 feet and seeing how many land in the inscribed circle of radius 1. Excited about this scheme to estimate \( \pi \) you line up in front of your poster, darts in hand. It suddenly it dawns on you that you can’t actually throw darts uniformly. They always seem to go to the right. The actual distribution of your dart throws is \( f(x,y) = x \) on \( 0 \leq x \leq 1, -1 \leq y \leq 1 \)

(a) If you throw \( n \) darts and count the fraction that are in the circle to estimate \( \pi \), what is the bias of your estimation?

(b) To correct for the fact that your throws are not uniform, instead of counting each dart equally you will need to count each dart proportionally to \( \frac{1}{f(x,y)} \). Given throws \((X_1,Y_1), \ldots, (X_n,Y_n)\) that are i.i.d \( f(x,y) = x \) on \( 0 \leq x \leq 1, -1 \leq y \leq 1 \), construct an unbiased estimator and show that it is indeed unbiased.

2. Suppose we have a circuit with voltage equal to \( \mu \). We have a voltage meter but its readings are uniformly distributed between \( \mu \) and \( \mu + 1 \). Since we know the readings are always greater than or equal to the true value, we decide to estimate \( \mu \) by taking the minimum value of our readings.

(a) Suppose we take two readings with our meter \( X_1 \) and \( X_2 \). What is the density function of our estimator \( \hat{\theta} = \min(X_1, X_2) \)

(b) What is the expected value \( E[\hat{\theta}] \)?

3. Suppose we want to understand how much a Brown student studies. We poll 1000 students and find that the average number of hours spent studying per week is 30 hours with standard deviation equal to 5.

(a) Give a 95% confidence interval for the study time.
(b) Give a 99% confidence interval for the study time.

4. A pollster would like to get an early start on the upcoming presidential primaries. The pollster wishes to estimate the proportion of the public who favor Clinton over Sanders. Suppose the pollster wants an error of estimation less than .04 with probability .9. How many people do they need to poll? How is this influenced by the true value of $p$?

5. Suppose $X$ is a binomial distribution with parameters $p$ and $n$. Find the Bias and MSE of the following estimators for $p$:

   (a) $\hat{\theta}_1 = \frac{X}{n}$

   (b) $\hat{\theta}_2 = \frac{X+1}{n+2}$

   (c) For which values of $p$ is $MSE(\hat{\theta}_1) < MSE(\hat{\theta}_2)$

6. Using real sources (newspaper, online sites) look at polls on the upcoming Rhode Island primaries. In particular, find the statistical reporting for the polls. What kind of sample sizes are used? What kind of margins of error are there? You do not need to write-up or turn anything in for this problem. Just look around and consider what you see.