Due before class on Friday, March 18th. It can be dropped off in the APMA 1650 homework box on the first floor of the APMA department, 182 George St OR at class (before it starts) on Friday.

APMA 1650: Complete all unstarrered problems.
APMA 1655: Complete all unstarrered and single starred problems. Double starred problems are particularly challenging and I do not necessarily expect you to answer them to completion. You should work on them and write up your (partial) solutions/ attempts/computations,
4. Suppose that the random variables \( Y_1 \) and \( Y_2 \) have joint probability distribution function \( f(y_1, y_2) \) given by

\[
f(x, y) = \begin{cases} 
6y_1^2y_2 & 0 \leq y_1 \leq y_2; y_1 + y_2 \leq 2 \\
0 & \text{elsewhere}
\end{cases}
\]

(a) Verify that this is a valid joint density function.
(b) What is the probability that \( Y_1 + Y_2 \) is less than 1?

5. (a) Suppose you have a lined piece of paper with parallel lines at distance \( d \) apart. Suppose you throw a small pencil of length \( L \), with \( L \leq d \) onto the surface uniformly at random. What is the probability that the pencil will intersect one of the lines?

(b) * Suppose you have a piece of paper with parallel vertical lines (distance \( d \) apart) and parallel horizontal lines (distance \( g \)) apart. Suppose you drop a small pen of length \( L \) (\( L \leq d \) and \( L \leq g \)) onto the surface uniformly at random. What is the probability that the pen will intersect at least one of the lines?

6. * Suppose \( X \) and \( Y \) are independent Binomial random variables with parameters \( (n, p) \) and \( (m, p) \) (same success probability \( p \) for both). Let \( Z = X + Y \).

(a) Show that \( Z \) is also a Binomial random variable. What are the parameters of this distribution?
(b) Show that the conditional probability distribution function of \( X \), given that \( Z = z \), is a hypergeometric distribution. What are the parameters of the distribution?

7. ** Suppose a stick of length 1 is broken in five places, each chosen uniformly at random along the length of the stick. Find the probability that the six resulting pieces can be arranged as the edges of a tetrahedron (triangular based pyramid). (This problem does not have a closed solution and might more accurately be labeled a 3 or 4 star problem. As always, give us your thoughts on how you would approach it.)