Due before class on Friday, Feb. 26th. It can be dropped off in the APMA 1650 homework box on the first floor of the APMA department, 182 George St OR at class (before it starts) on Friday.

APMA 1650: Complete all unstarred problems.
APMA 1655: Complete all unstarred and single starred problems.
Double starred problems are particularly challenging and I do not necessarily expect you to answer them to completion. You should work on them and write up your (partial) solutions/attempts/computations, etc.

Show all work!

1. Let $X$ be a geometric random variable with probability of success (on a single trial) equal to $\frac{1}{4}$, what is the probability that $X$ equals an odd integer?

2. Suppose there are $n$ students turning in homework to $m$ dropoff boxes. Each student puts their assignment in one of the boxes equally likely at random. What is the expected number of dropoff boxes that are empty?

3. Using Markov’s inequality, bound the probability of:
   
   (a) Flipping $\leq 10$ Heads from 100 flips of a fair coin.
   (b) Flipping $\leq 10$ Heads from 100 flips of a biased coin with $p(\text{Heads}) = \frac{1}{5}$.

   Using Chebyshev’s inequality, bound the probability of:
   
   (c) Flipping between 40 and 60 Heads from 100 flips of a fair coin.
   (d) Flipping between 10 and 30 Heads from 100 flips of a biased coin with $p(\text{Heads}) = \frac{1}{5}$.

4. Let $X$ be a random variable, $V(X)$ be the variance, and $\alpha$ and $\beta$ be constants. Show that

   $$V(\beta X + \alpha) = \beta^2 V(X)$$

   (To think about: what is the variance of a constant function? Why does that make sense?)

5. * Let $X$ be a random variable, define the random variable

   $$Y = \frac{X - E(X)}{\sqrt{V(X)}}$$

   What is the expectation and variance of $Y$? Give purely numerical answers.
6. (a) Will places star stickers on graded homeworks with a Poisson distribution with parameter $\lambda_1 = 20$ over the entire homework grading period. He starts with a sheet of 30 stickers. What is the probability that he runs out of stickers during the grading period? (You do not need to fully calculate the sum; just write the correct formula.)

(b) Karen places elephant stickers (which are more expensive) on graded homework with a Poisson distribution with parameter $\lambda_2 = 10$. She has only 15 elephant stickers. What is the probability that she runs out?

(c) What is the total expected number of stickers (of any type) given out?

(d) What is the probability that no stickers (exactly 0 of either type) are given out?