Mesh Generation With Large Deformations and Topological Changes

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Tracking Time-Dependent Domains
Free boundary problems arise in many areas of mathematics, physics, and engineering. Understanding free surface dynamics is important for applications such as coating flows [4], simulating water wave dynamics for computer graphics [9], and surface tension/curvature driven flows in microfluidic devices such as Holo-Shake flow [7, 10, 16, 14]. Other examples involve fluid-structure interactions [13, 17, 4]. However, in any application with a moving boundary, the deformation of the domain is the main obstacle in obtaining a tractable physical model. In addition, some of these applications exhibit topological changes (i.e. pinching or joining of disjoint parts of the interface) and prove even more difficult to model [3, 6, 15, 8].

Main Ideas
- FEM is a flexible tool to solve many kinds of multi-physics problems.
- General domain shapes can be considered, but generating a mesh for an arbitrary domain can be expensive. This issue is further exacerbated in free boundary problems.

Large Deformations and Topological Changes

- Level Set Methods:
  - Advantages: Eulerian Grid. Automatic handling of topological changes.
- Variational/Front Tracking:
  - Advantages: Geometry is accurately represented. Mass is conserved.
  - Issues: Mesh distortion and remeshing.

Mesh Generation

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Distance Functions And Shape Skeletons

- Compute the signed distance function $\phi$ on the previous unstructured triangular mesh. $O(N_t^{1/4})$ operations. Possibility of parallelizing.
- Straightforward processing of $\phi$ gives an approximation of the shape skeleton, i.e. look for jumps in $\Phi_0$ $O(N_t + N_E + N_P)$ operations.
- Let $\Gamma_{Sk}$ be the approximate shape skeleton.
- Crucial step: compute the distance function $\zeta$ (to $\Gamma_{Sk}$) on the previous mesh.

Adaptive Refinement

- Interpolate distance functions $\phi$ and $\zeta$ onto the new mesh vertices. Existence searching can be avoided by taking advantage of the sequentially refined grid.
- Adaptively refine (by Rivara bisection) all triangles until no triangle overlaps both $\Gamma$ and $\Gamma_{Sk}$.
- This is an $O(N_t^{1/4})$ algorithm and is guaranteed to terminate for smooth manifolds. Resulting grid resolves shape topology, e.g. fingering.
- Further refinement achieved via curvature estimation; $O(N_t^{1/2}(\Gamma))$.

Candidate Manifold Selection

- Interior and exterior triangles are selected via the distance function. This produces a candidate manifold of edge segments, $O(N_E^{1/2})$. This is the most critical part.
- Even with minor pruning, this will introduce an aliasing effect.
- Mesh conformity is ensured via a shape optimization technique; $O(N_t^{1/2}(\Gamma))$.

Results

- EWOD driven droplet splitting, only the interior mesh is shown.
- The droplet pinches in two places (symmetrically) resulting in an elongated satellite droplet.

Extension To 3D
Selecting The Candidate Manifold

- Everything generalizes to 3-D, except the candidate manifold selection process. Must ensure that no tetrahedra will get crushed during the mesh conforming phase.
- This will require a more complicated manifold selection process, which may include edge/face swapping to give a well-formed polyhedral surface.

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