ANDREW GLEASON

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ANDREW MATTEI GLEASON was elected to the American Philosophical Society in 1977. Gleason, who was a great mathematician, a grand teacher and mentor, and a recipient of many honors, was once described as "one of the quiet giants of twentieth-century mathematics."[1]

His direct engagement on an intellectual level with everyone with whom he came into contact was legendary. "Have you ever thought of this?" is how Andrew Gleason often preceded the formulation of some idea, or some question, to his mathematical colleagues and students. Usually we hadn't (thought of the idea) and even if we had we would not have expressed it in as clarifying, or as enticing, a manner as he did. The "this" could range quite broadly: ideas related to transformation groups and his famous solution to Hilbert's Fifth Problem, to measure theory, projective geometry, Hilbert Spaces, or to combinatorics, to graph theory, to coding theory, or—and this was also one of Gleason's many great loves—to the teaching and perfection of mathematical skills at any level (how to treat measuring when teaching first-graders; re-forming the teaching of calculus; savoring the latest Putnam Competition exam questions). Quite a span of mathematics.

And his other interests were broad. His wife, Jean Berko Gleason, recalled that "he loved looking at the stars. He knew every star in the sky and could tell you their names. Early on, he was planning on becoming an astronomer, but then he learned how cold it was to sit outside and watch."

Andrew Gleason's own early education had a significant geographical span; he graduated from high school in Yonkers, New York, having also taken courses in Berkeley, California. His undergraduate years at Yale were spent largely taking graduate-level courses. When Andy graduated in 1942, he joined the U.S. Navy as a member of a group of eight to ten mathematicians working to crack enemy codes.[2]

In 1946 Gleason came to Harvard, having been elected (as a junior fellow) to the Society of Fellows. To be such a fellow in those days meant that one could achieve an academic career without having a Ph.D. Andy never did pursue a doctoral degree, his highest degree being an M.A. awarded to him by Harvard in 1953. He spent his time in the Society

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[1] This is from Ethan Bolker's "Andrew M. Gleason 1921–2008," which is a marvelous collection of memoirs on Andy Gleason's life and works (*Notices of the American Mathematical Society* 56.10 [November 2009]:1236–67).

[2] For the (declassified) story of Andy's contribution to the code-breaking effort—including work on the German Enigma ciphers—and for a description of his important lifelong engagement in cryptography studies for the government, see "The Secret Life of Andy Gleason" by Burroughs, Lieberman, and Reeds (http://www.cs.umb.edu/~eb/gleason/lieberman11.pdf).

of Fellows developing a broad mathematical culture and thinking about Hilbert's Fifth Problem.

At the end of his three-year fellowship Gleason was appointed assistant professor in the Harvard Department of Mathematics. Soon thereafter, he took a two-year leave of absence from Harvard to return to the U.S. Navy to serve during the Korean War (mid-1950 to mid-1953). After this, Gleason returned to Harvard, where he spent the rest of his academic career. He was appointed full professor in 1957.

Gleason married Jean Berko in 1959. Jean Berko Gleason, a prominent psycholinguist, has had a distinguished academic career as a professor in the Department of Psychology at Boston University. The Gleasons had three daughters, Katherine, Pamela, and Cynthia.

Gleason was named the Hollis Professor of Mathematicks and Natural Philosophy in 1969 (this being the oldest endowed chair in the sciences in the United States). He became a senior fellow of the Harvard Society of Fellows in 1977 and was chair of the Society of Fellows from 1989 to 1996. Gleason retired from Harvard University in 1992.

His mathematical conversations, his seminar discussions, his writing, and his lectures had these qualities, most cherished in a mathematician: he was comprehensible, clear, and to the point; his formulations had a scintillating precision, and they were always delivered with enthusiasm and wide-eyed wonder. One of his colleagues once summed this up by saying, "When he touched a thing, he made it shine." Ethan Bolker, who studied with him, wrote that Gleason "took such care preparing, and searched so hard for economy and elegance" in his lectures that "the rough places were made plain. The hard parts didn't seem so in the seamless flow, so it could be hard to find the crux of a proof."

Andy's devotion to the intellectual community and his sense of service were extraordinary. His late colleague Raoul Bott once joked that Andy lacked the essential Hungarian talent for being absent when an important administrative task needed to be done. Benedict Gross once said, "Andy served Harvard, his department, the Society of Fellows, and the mathematical profession with generosity and skill. Although he held strong opinions, he never imposed them on others, and never made anyone feel small if they didn't possess his brilliance. In fact, I never heard Andy raise his voice, either in conversation or in a meeting. That's not to say he wasn't convincing—it was his vision of a small faculty, training the best graduate students and amplified by the energy of outstanding undergraduates, that defines the mathematics department we have at Harvard today."

Gleason's best-known work is his resolution of Hilbert's Fifth Problem. David Hilbert, slightly over a century ago, formulated two dozen problems that have—since then—represented celebrated milestones

measuring mathematical progress. Many of the advances in Hilbert's problems initiate whole new fields, new viewpoints. Those few mathematicians who have resolved one of these problems have been referred to as members of the "honors class."

Hilbert fashioned his "Fifth Problem" as a way of offering a general commodious context for the then-new theory of Sophus Lie regarding transformation groups. Nowadays, Lie's theory is the mainstay of much mathematics and physics, and his kind of groups, Lie groups, constitute an important feature of our basic scientific landscape.

One can think of a transformation group as a collection of symmetries of a geometric space. Some spaces admit infinitely many symmetries: think of the circle, which can be rotated at any angle. The grand problem facing Sophus Lie is how to deal with these infinite groups of symmetries. Given Lie's theory, it is natural to restrict attention to situations where one can use the methods of the calculus to effectively treat the issues that arise in connection with these infinite transformation groups. The brilliance of Lie's approach is that—given the hypothesis that one can apply the techniques of the calculus—one reduces many of the thorny questions about these groups to essentially linear problems.

Hilbert raised the question, in effect, of how restrictive this hypothesis is; i.e., how far Lie's concept of continuous groups of transformations is approachable without the assumption of the differentiability of the functions defining the transformations of group. At the International Congress of Mathematicians held in Cambridge in 1950, Andy proposed a possible method to arrive at a happy answer to this question; the answer being, roughly speaking, quite far.

Andy emphasized the central role played by the one-parameter subgroups in the picture. And he focused on a condition that has been the mainstay of the theory of continuous transformation groups ever since: the condition that a group have no small subgroups.

The following year Gleason proved a key result about maximal connected compact subgroups. The year after that, using results of Montgomery and Zippin, and Yamabe, Andy clinched things, resolving Hilbert's question. An extremely important advance.

The depth of Andy's work is extraordinary, as is its breadth: from his computer explorations very early in the history of machine computation (a search problem in the n-cube) to solving a conjecture of his late colleague George Mackey (about measures on the closed subspaces of a Hilbert space) to the intricacies of finite projective geometry and coding theory, to the relationship between complex analytic geometry and Banach algebras. Gleason was also one of the rare breed of mathematicians who don't stay on just one side of the pure mathematics/applied mathematics "divide." In fact, his work and attitude gave testimony to

the tenet that there is no essential divide. Indeed, the ideas and mathematical interests that Andy nurtured in his applied work for the government, which was a passion for him throughout his lifetime, connect well with his public work on finite geometries, and his love for combinatorics.

Andy's interest in the training of mathematicians, and in exposition and teaching in general, led him to edit with co-authors a compendium of three decades of William Lowell Putnam mathematical competition problems, led him to write a bold text formulating the foundations of analysis starting with a grand and lucid tour of logic and set theory, and also led him to engage in the important project of K–12 mathematical education, and reform efforts in the teaching of calculus.

The founding idea behind the various mathematical education initiatives with which Andy was involved—either the programs for early mathematical education that were referred to (by both detractors and promoters) as New Math, or the programs for teaching calculus (providing syllabi and texts that came to be referred to as the Harvard Consortium)—was to present mathematics concretely and intuitively, and to energize and empower the students and teachers. Andy's credo for this program as for all of his teaching was that the ideas should be based in equal parts of geometry for visualization of the concepts, computation for grounding in the real world, and algebraic manipulation for power.

This relates to Andy's general view: a working mathematician should have at his or her disposal a toolkit of basic techniques for analyzing any problem. He felt that all good problems in math—at any level—should weave together algebra, geometry, and analysis, and students must learn to draw on any of these tools, having them all "at the ready." Andy emphasized this in his department's discussion regarding the structure of the comprehensive qualifying exam for graduate students. He also loved to think about exam problems that exhibited this unifying call upon different techniques; for example, he would work out the problems of the (undergraduate) Putnam Competition exam, year after year, just for fun.

Andy had many honors. He received the Newcomb Cleveland Prize from the American Association for the Advancement of Science for his work on Hilbert's Fifth Problem. He received the Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics, the Mathematical Association of America's most prestigious award. He was president of the American Mathematical Society (1981–82) and a member of the National Academy of Sciences, the American Academy of Arts and Sciences, and—naturally—the American Philosophical Society.

Elected 1977; Committees: Advisory on Election of Members 1999–2002; Membership I 1997–2002

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