APMA 0360: HOMEWORK ASSIGNMENT #7
DUE DATE: 4PM, OCT 26TH, 2012

Name: Grade:

Section 9.3
For the problems in this section, you may use "stability/instability table 9.3.1", page 513, when needed, to draw your final conclusions.

- Work out problems 1, 3, 5, 7, 13. Note that in the question (d) of problems 5,7,and 13, try as much as possible to sketch a phase portrait at least locally near the critical point(s). You may use computers to draw these if you know how to do so, but not required.
- Work out problem 21 parts (a), (b), and (c), and problems 27 & 28.

Stability/instability questions

Lemma 1. (Stability) *Let* $\mathbf{x_0}$ *be a critical point of*

$$\mathbf{x'(t) = f(x)}, \qquad \mathbf{t \geq 0}.$$

Assume that there are two positive constants C_0, δ_0 *and a nonnegative constant* α *so that whenever* $\mathbf{x(0)}$ *satisfies* $\|\mathbf{x(0)} - \mathbf{x_0}\| \leq \delta_0$*, the solution* $\mathbf{x(t)}$ *with the initial data* $\mathbf{x(0)}$ *exists for all time* $t \geq 0$ *and satisfies*

$$\|\mathbf{x(t)} - \mathbf{x_0}\| \quad \leq \quad \mathbf{C_0 e^{-\alpha t}}\|\mathbf{x(0)} - \mathbf{x_0}\|, \qquad \forall\ \mathbf{t \geq 0}.$$

Then **prove by the stability definition** *given in the book or in class that the critical point* $\mathbf{x_0}$ *is stable if* $\alpha \geq 0$ *and asymptotically stable if* $\alpha > 0$.

- Once you have proved the above lemma, it might serve as a definition of stability of the critical point throughout the class.

- Show that the critical point $x = 0$ of each of the following systems of differential equations is (asymptotically) stable or unstable. In this problem, it's asked you to **show your proof** without using the "stability/instability table 9.3.1". You may use the above lemma, that is to verify the assumptions of the lemma for each of the systems:

(a) $\begin{cases} x'(t) = -x + \sin y + y^3 \\ y'(t) = -\sin x - y \end{cases}$ (b) $\begin{cases} x'(t) = 2x - x^2 - xy \\ y'(t) = -y - 2y^2 + x^2 y \end{cases}$