APMA 0360: HOMEWORK ASSIGNMENT #3

DUE DATE: 4PM, SEP 28TH, 2012

Name: Grade:

Exercise 1: In each of the following systems of differential equations,

- find the general **real-valued** solutions,
- sketch a phase portrait with a few trajectories and discuss its behavior as the time $t \to +\infty$.
- determine the type of the origin (that is, a saddle point, stable/unstable node, proper/improper node, stable/unstable spiral, or center).

(a) $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$ (d) $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \mathbf{x}$

Exercise 2: Solve the following initial value problem:

(a) $\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(\mathbf{0}) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(b) $\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(\mathbf{0}) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Exercise 3: Let A be an arbitrary square real-valued matrix and let λ and X be an eigenvalue and an associated eigenvector (that is, $AX = \lambda X$). Assume that λ is a complex number with its nonzero imaginary part. Show that

- Show that the complex conjugate $\bar{\lambda}$ is also an eigenvalue. Find a eigenvector associated with $\bar{\lambda}$.
- If we write $X = u + iv$ with u and v being real, then the two vectors u and v must be linearly independent (Hint: you may use Problem 34, Section 7.3).

Exercise 4: Problem 28, a, b, c, Section 7.5

Exercise 5: Consider the system

$$\mathbf{x}' = \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix} \mathbf{x}$$

with $\alpha \in \mathbb{R}$.

a) Determine the eigenvalues and eigenvectors in terms of α.
b) Find the critical value of α where the qualitative nature of the phase portrait for the system changes.
c) Draw a phase portrait for a value of α slightly below, and for another value slightly above, the critical value.

Exercise 6: A mass m on a spring with constant k satisfies the differential equation:

$$mu'' + ku = 0,$$

where $u(t)$ is the displacement at the time t of the mass from its equilibrium position.

(a) Write the above differential equation in the form of first order differential equations:

$$X'(t) = AX(t).$$

(b) Find the eigenvalues, assuming that both m and k are positive numbers.
(c) Find the general solutions and sketch the phase plane when $m = 1$ and $k = 1, 4, 9$.