

APMA 0360: HOMEWORK ASSIGNMENT #2

DUE DATE: SEP 21ST, 2011

Name:

Grade:

**EXERCISE 1**

Using the Gaussian elimination method to solve the following systems of linear algebraic equations (or showing that there is no solution if it is the case):

$$(a) \quad \begin{cases} x_1 + x_2 + x_3 = 2, \\ 2x_1 + x_2 = 1, \\ x_1 - 2x_2 - 5x_3 = 0. \end{cases} \quad (b) \quad \begin{cases} x_1 - 2x_2 + 2x_3 = 0, \\ 2x_1 + x_2 - x_3 = 1, \\ -2x_2 - 4x_3 = 0. \end{cases}$$

**EXERCISE 2**

Find all the eigenvalues and compute the eigenvectors associated with each eigenvalue of the following matrices:

$$(a) \quad \mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (b) \quad \mathbf{B} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, \quad (c) \quad \mathbf{C} = \begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix},$$
$$(d) \quad \mathbf{D} = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}.$$

**EXERCISE 3**

Consider the system

$$\mathbf{x}' = \mathbf{A}\mathbf{x}, \quad \text{with} \quad \mathbf{A} := \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$$

- Compute the eigenvalues and eigenvectors of the matrix  $\mathbf{A}$ .
- Find the general solution of the above equation.

**EXERCISE 4**

Consider the system

$$\mathbf{x}' = \mathbf{A}\mathbf{x}, \quad \text{with} \quad \mathbf{A} := \begin{pmatrix} -3 & -2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- Compute the eigenvalues and eigenvectors of the above matrix.
- Find the general solution of the above equation.

**EXERCISES from the textbook**

- Section 7.3: Complete the Problems 26 and 32. Show your work. There,  $A^T$  denotes the transpose matrix of  $A$ , and  $A^*$  denotes the adjoint of  $A$  and is defined by taking the transpose and complex conjugate of  $A$ . A matrix is called Hermitian or self-adjoint if  $A^* = A$ .
- Section 7.3: Problem 34.
- Section: 7.4: Solve Problem 2 with  $n = 2$  (that is, do only part a,b, and c in this problem).