

Review Problems

I. Solve the following PDE in a disk: $\{x^2 + y^2 \leq 1\}$

$$u_{xx} + u_{yy} = 0.$$

The boundary condition in the polar coordinates (r, θ) is $u(r, \theta)|_{r=1} = \sin \theta$.

II. Solve the following PDE in the interval: $0 \leq (x, y) \leq 1$

$$\begin{aligned}u_{xx} + u_{yy} &= 0, \\u(x, 0) &= u(0, y) = u(1, y) = 0, \\u_y(x, 1) &= \sin(2\pi x)\end{aligned}$$

III. Solve the following PDE in the interval: $0 \leq x \leq 2, t \geq 0$

$$\begin{aligned}u_{tt} - 4u_{xx} &= 0, \\u_x(0, t) &= 0, u_x(2, t) = 0, \\u(0, x) &= f(x), u_t(x, 0) = 0.\end{aligned}$$

IV. Solve the following PDE in the interval: $0 \leq x \leq 2, t \geq 0$

$$\begin{aligned}u_t - 4u_{xx} &= 0, \\u_x(0, t) &= 0, u(2, t) = 0, \\u(0, x) &= f(x)\end{aligned}$$

V. write down the Forward Euler, Backward Euler, and Improved Euler formula for the following equations:

$$x' = 3 - t + 4x.$$

VI. For the following equations:

1.

$$x' = 14x - 2x^2 - xy \quad y' = 16y - 2y^2 - xy$$

2.

$$x' = -x + \sin y \quad y' = 2y$$

3.

$$x' = x(3 - y) \quad y' = y(-2 + x)$$

4.

$$x'' + 9x - x^3 = 0$$

- a Determine all the critical points.
- b Find the linear equation for those critical points.
- c Determine nonlinear stability properties of the equations in the neighborhood of these critical points.
- d Sketch phase portraits in the neighborhood of these critical points.

VII. Please review the midterm, similar problems may appear in the final!