HW5 Solutions

1. (a) The masses of an electron and positron are both $M_e = 9 \times 10^{-31}$ Kg. If they are at rest the initial energy is

$$E_i = 2M_e c^2 \approx (18 \times 10^{-31} Kg)(9 \times 10^{16} m^2/s^2) \approx 10^{-13} J$$

By conservation of mass-energy $E_i = E_f$, which is the total energy carried away by the two photons.

While not asked in the problem, one might wonder how E_f relates to each individual photon's energy E_1 and E_2? By conservation of momentum each photon's momentum is equal and opposite amounts (because the initial momentum is zero). Since the magnitude of relativistic momentum of photon 1 and 2 are equal $P_1 = P_2$ so are their energies $E_1 = E_2$. That is, each photon will carry away half of E_i.

(b) 26.9 million Joules is 2.69×10^7 Joules. From the calculation of part a, we estimate about

$$\left(2.69 \times 10^7\right) / 10^{-13} \approx \times 10^{20}$$

electron-positron annihilations are necessary, and around 10^{20} positrons in total. The mass of a single positron (again, from part a) is 9×10^{-31} Kg, and so we need a supply of positrons whose mass is

$$(9 \times 10^{-31}\text{Kg}) \times (10^{20}) \approx 9 \times 10^{-11}\text{Kg} \approx 10^{-10}\text{Kg}\,.$$

According to NASA, a well-designed machine can produce "10 milligrams of positrons for [...] about 250 million dollars". Noting that 10 milligrams is 10^{-5} Kg, the cost is about

$$\frac{\$250,000,000}{10^{-5}\text{Kg}} \times 10^{-10}\text{Kg} \approx \$250,000,000 \times 10^{-5} \approx \$2,500\,.$$

2.

3. Start by drawing a figure and label the angle made by the two lines meeting at the north pole by ϕ. Since the other two angles are 90 degrees each (equivalently $\pi/2$ each) the sum of Angles is

$$\text{Angles} = \pi + \phi$$

Next, recall that the area of a ball of radius r is $4\pi r^2$. So, as a special case, whenever $\phi = 2\pi$ the entire northern hemisphere of the ball is "covered" by our triangle – the triangle's area would be $2\pi r^2$. If $\phi = 0$ the area would be 0. With a little thought we see the triangle's area to be

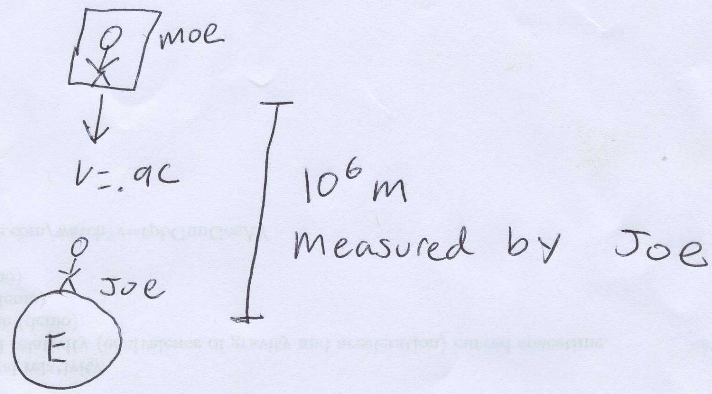
$$\text{Area} = \frac{4\pi r^2}{2(2\pi/\phi)} = \phi r^2$$

From the angle and area formulas we have

$$\text{Angles} = \pi + \frac{\text{Area}}{r^2}$$

HW4

(1) a)



b)

$$L_{moe} = L_{Joe}\sqrt{1-\frac{v^2}{c^2}} = L_{Joe}\sqrt{1-.81} \sim 4 \times 10^5 \text{ m}$$

To moe Earth is approaching at .9c, hence the distance is contracted.