Theory of Relativity – Final Quiz
July 10, 2014

Name:____

Below are short questions and problems. Answer to the best of your ability.

VERY short answers. Each worth 1 point.

1. What is the principle of relativity? Name the three versions of the principle of relativity (developed in the 1600s, 1905 and 1916) discussed in this course.

The laws of physics are the same in all inertial reference frames. There are three versions: Galilean, special and general.

2. A ship is moving near the speed of light and fires a laser beam. According to special relativity what will a stationary observer measure the laser beam's speed to be? What will an observer located on the ship measure the laser beam's speed to be?

Both will measure the speed of light to be $c = 3 \times 10^8$ m/s

3. Suppose I'm an inertial observer and you move at a speed $v = 10m/s$ relative to me. Are you an inertial observer?

Yes

4. List 3 ways our life would be different if the speed of light were $80km/hour$.

You would age appreciably less after a bike ride. Cars would shrink in the direction of motion. Can't talk of friends across the country in "real-time". Can't go faster than 80.

5. What is the resolution to the twin paradox?

Suppose twin A accelerates to compare watches. This twin is no longer an inertial observer (they know they are moving). The other twin (B) is always inertial and so their calculations are correct.

6. Suppose the Michaelson-Morley experiment discovered aether. If the aether wind moved at exactly the speed of light and you fired a laser against the aether wind what would light's speed be? (Hint: this is before special relativity physics, think of the airplane problem).

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7. Why does the Foucault pendulum appear to rotate throughout the day?

Earth is rotating on its axis - Earth is a non-inertial reference frame.

8. According to special relativity, if you continue to push a ball by applying a constant force at some point you can hardly increase the velocity, yet you continue to do work on the ball. Where does this extra energy go?

Into its inertial mass.

9. A consequence of the equivalence principles is that light bends in a gravitational field. What startling suggestion did this lead Einstein to make about our spacetime geometry?

Its non-Euclidean (curved spacetime).

10. The equation $E = m_0c^2$ is often given in popular accounts of special relativity. What is missing from this equation? Please write either the correct equation or describe with words what "feature" has been neglected.

No. Its missing the kinetic energy part (equivalently $E = \gamma m_0 c^2$).

11. Suppose you take a long voyage to a distant star and return home. You are paid at an hourly rate. Will you be paid more going by the Earth's clock or the ship's clock?

Earth

12. A farmer has a ladder which is too big for his barn. So one day he ran with the ladder as fast as he could - the ladder having Lorentz contracted to a size the barn could easily accommodate. Once the back end of the ladder was fully inside the barn a friend would slam the door behind the farmer and the farmer would come to a complete stop. However, the friend points out that in the farmers reference frame the barn, not the ladder, will contract and so the fit will be even worse. Who is correct and why?

Simple explanation: the ladder must accelerate to stop, and so the barn's frame is correct.

Problems. Please attempt at least one. Extra credit for more. 2 points each

1. In Back to the Future 4 (the unreleased sequel) Marty goes 500 seconds into Earth's future in what seems to be 300 seconds to him by moving at $v = 140km/hour$. According to special relativity what would the speed of light need to be in the Back to the Future universe for this to happen? You can leave your answer expressed as some number times v.

From the time dilation formula we have

$$500s = \gamma(300s) \quad \rightarrow \quad \gamma = 5/3$$

Solving for the (Back to the future) speed of light

$$\begin{aligned} 5/3 = \gamma &= \frac{1}{\sqrt{1 - v^2/c^2}} \\ \sqrt{1 - v^2/c^2} &= 3/5 \\ 1 - v^2/c^2 &= 9/25 \\ v^2/c^2 &= 16/25 \end{aligned}$$

And so $c = 5/4v = 175km/hour = 48m/s$

2. Suppose Nolan Ryan could throw a fastball at $v = (4/5)c$. If a baseball's rest mass is $1kg$, what is the ball's mass while its in motion? What is its relativistic kinetic energy? How does this compare to the usual $\frac{1}{2}m_0v^2$ expression for the kinetic energy (greater, less equal)? For this problem you might need the formula $E = \gamma m_0 c^2 = m_0c^2+$(relativistic kinetic energy).

$v = (4/5)c$, and so by the previous problem $\gamma = 5/3$. $M = m_0\gamma = 5/3$ kg.
Relativistic kinetic energy is $(m_0\gamma - m_0)c^2 = \frac{2}{3}m_0c^2$
Classical kinetic energy is $\frac{1}{2}m_0v^2 = \frac{1}{2}m_0(\frac{4}{5}c)^2 = \frac{8}{25}m_0c^2$
Relativistic kinetic energy is greater.

3. A sailboat is manufactured so that the mast leans at an angle θ' with respect to the deck. An observer standing on a dock sees the boat go by at a speed $v = .9c$. What angle does this observer say the mast makes?

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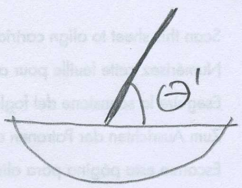

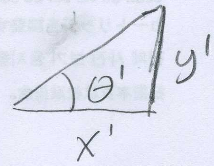

$$\tan\theta' = \frac{y'}{x'}$$

Lorentz contraction in x only

$$y = y'$$

$$x = \frac{x'}{\gamma} \implies x' = \gamma x$$

and so

$$\tan\theta' = \frac{y'}{x'} = \frac{y}{\gamma x} = \frac{1}{\gamma}\left(\frac{y}{x}\right) = \gamma^{-1}\tan\theta$$

so,

$$\tan\theta = \gamma \tan\theta'$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{9}{10}\right)^2}} \approx 2.3$$

$$\tan\theta = (2.3)\tan\theta'$$