Brown/Paris Numerical Analysis: Problem set 7

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1 Shooting methods for boundary value problems

In the first lectures we already learned how to use the finite difference method to solve the Poisson equation

$$\partial_{xx} T = f \tag{1}$$
$$T(0) = 0 \tag{2}$$
$$T(L) = 1 \tag{3}$$

and saw that it required the solution of a linear system of equations. Today we will learn how we can use a combination of an ODE solver and a non-linear root solver to find the solution to this equation.

- First convert the PDE to a system of first order equations
- Set up your favorite ODE solver (explicit Euler,RK3,etc) to solve the ODE from $x = 0$ to $x = L$ with initial conditions $T(0) = 0$ and $T'(0) = \alpha$ where α is a constant taken as an input parameter.
- We can now define a function $f(\alpha)$ that returns the value of $T(L)$. If we want to find α such that T is faithful to the boundary conditions of the PDE, we can solve the equation

  $$g(\alpha) = f(\alpha) - 1$$

  using our favorite root finding method.

Use the shooting method to solve the Poisson equation for $L = \pi/2$ and $f = -sin(x)$ and compare the efficiency of this approach to the methods we looked at in Lecture 1