Brown/Paris Numerical Analysis: Problem set 6

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1 Symplectic methods for integrating ODEs

Since everybody has some background numerically solving initial value problems already, we're going to look at a topic in numerical ODE in a little more depth. For this assignment we're going to simulate the motion of Earth's orbit around the sun.

To a good approximation, all planets obey Newton's law of gravitation:

$$F_{ij} = G\frac{m_i m_j}{r_{ij}^2}$$

where G is the gravitational constant. The motion of each planet is governed by Newton's second law:

$$m_i \ddot{x}_i = \sum_j F_{ij}$$

Problem one:
We are going to first determine how different choices of scheme for solving the governing ODE will affect the orbit of the Earth around the Sun when the presence of all other planets are neglected.

- Use the forward Euler, backward Euler, and the symplectic Euler method to solve the governing equation for one orbit.
- How do these methods behave qualitatively? How does this trend vary when you change the timestep? How can you interpret this in terms of the amount of energy in the system?
- You already have some experience with Runge-Kutta methods; what happens when you try to solve the ODE with a high order Runge-Kutta scheme?

After completing this problem we will break for a bit of lecture to explain formally what is going on.

Problem two:
Now that we have a sense of going on, let's simulate the solar system (or at least part of it). For ease of implementation, feel free to only simulate some of the planets (but do at least 4).

Useful information:

- $G = 6.673 \times 10^{-11}\ N \cdot m^2/kg^2$
- `https://en.wikipedia.org/wiki/List_of_Solar_System_objects_by_size`
- `http://www.sjsu.edu/faculty/watkins/orbital.htm`