AM119: Assignment 1 - (Due beginning of class 2/29)

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1 Finite difference solution of the 1D periodic advection-diffusion equation

Today we learned the basics of the finite difference method. We'll use this method as an exercise to pull together some of the C++ we've learned. On the course website, you'll find a zip file with code snippets. As discussed in class, these examples have all the C/C++ that you'll need to complete this assignment. Make sure that before you start this assignment, you completely understand all of these examples. You will find a directory called finiteDifference in the same zip file - use the sample code there as a starting point for this assignment.

Goal: We seek a numerical solution of the advection-diffusion equation.

$$\partial_t u + a\partial_x u - \nu \partial_{xx} u = f$$

with periodic boundary conditions

$$u(0,t) = u(2\pi, t)$$

and initial condition

$$u(x,0) = \sin(x)$$

- Apply to each term in the PDE (from left to right): a forward difference formula in time, a backward difference formula in space, and a centered difference formula to obtain the discrete equation

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + a\frac{u_i^n - u_{i-1}^n}{\Delta x} - \nu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} = f_i$$

- Use Taylor series to demonstrate that this formula is accurate at each node to order $O(\Delta x + \Delta t)$.
- For each i, solve for u_i^{n+1} to obtain a formula for updating our discrete solution at each timestep
- Write a C++ program using this formula to march forward in time until $t = 1$, and compare to the exact solution. For parameters, take $a = \nu = 1$, and choose Δx and Δt so that $\frac{a\Delta t}{\Delta x} < \frac{1}{2}$ and $\frac{\nu \Delta t}{\Delta x^2} < \frac{1}{4}$.

- Convince me that your code works - provide evidence that for more points and for smaller timesteps your code converges to the exact solution.

When turning in this, and other assignments, make sure to leave a functioning copy of the code that you've run in your data directory. Prepare a 1-2 page short report demonstrating your results with properly labeled plots/tables, and submit to me via email with the subject line APMA119-YourName-HWXX. It is not necessary to print/email me any code - I will look into your data directory and make sure that your code backs up your claim. Please make sure to include in your report the directory and filename where I can find your code.