Art, Mathematics and the *Zeitgeist:* parallels between the two most international disciplines

David Mumford

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OUTLINE

- Why do math and art have anything to do with each other?
- Jackson Pollock and the Atomic Bomb
- Early 19th century: Breaking the ties with the concrete and tangible
- Late 19th century: Playing with the components of our models of reality, with the ‘ugly’.
- 20th century: Full blown abstraction

- The saga of mathematics is unknown outside a narrow coterie; the high points of art are basic ingredients of a liberal education. Can we use our knowledge of the latter to open up the former?
- There are stunning parallels between the development of the two. WHY?? Both fell in love with the abstract. But there are few examples of direct contact/influence.
- Somehow, the *zeitgeist* seeps over the levees.
- Neither art nor math respects national boundaries; they are international languages.
- I want to use these links to explain to the math-phobic something of what has been going on in math.

A first example of the uncanny Art/Math linkage, from post WWII America

<u>Beauty and power through randomness</u>

The discovery that randomness can be harnessed to create science and beauty, c.1945

Jackson Pollock, *Autumn Rhythm, #30*, 1950

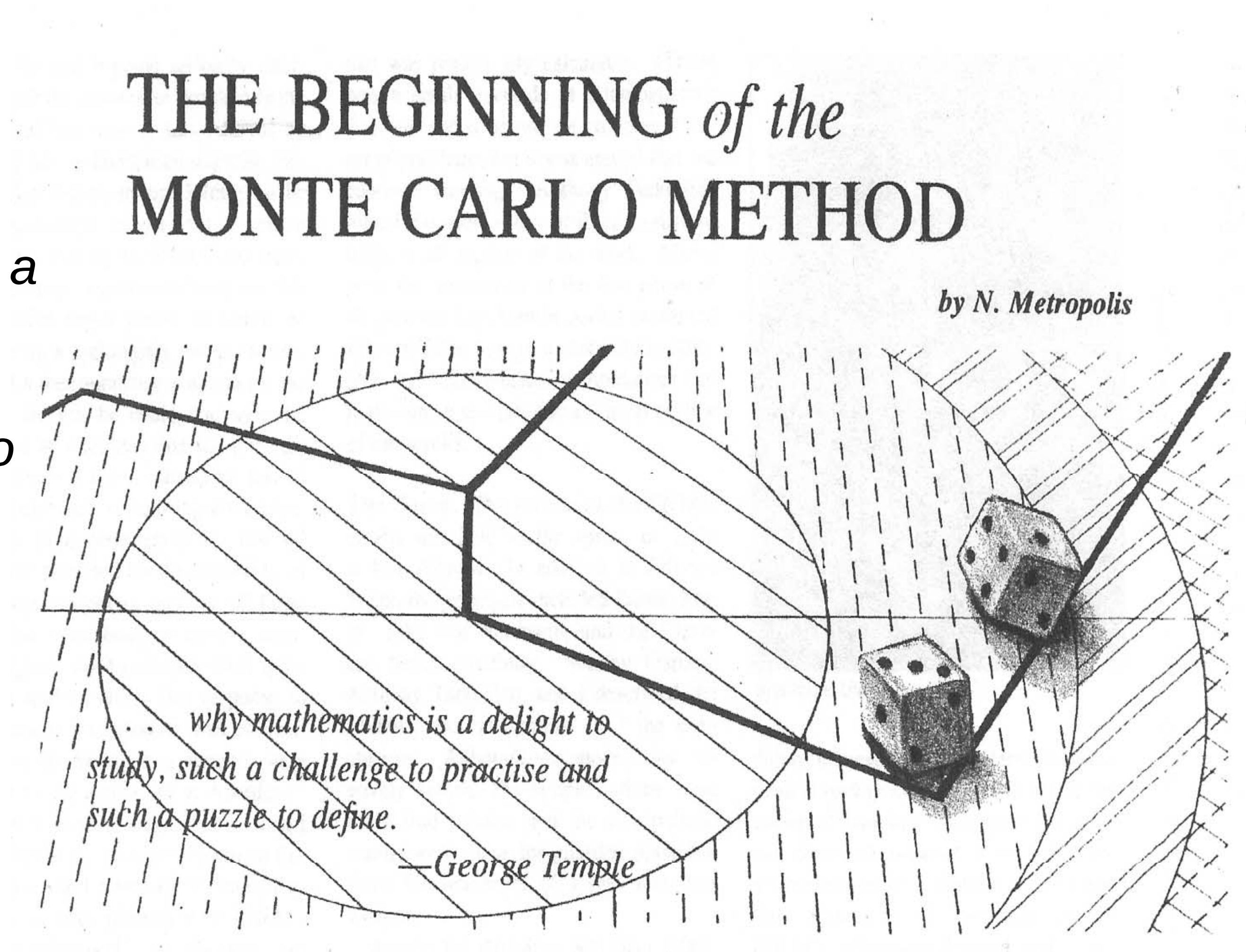
Pollock's "Action Painting"

- When the German emigré artist and intellectual, Hans Hoffmann, suggested to Jackson Pollock that he "observe nature" or his painting would become repetitious, Pollock – born in Cody, Wyoming – famously responded "f*** you, I am nature."
- "Strict control is what Pollock gave up when he began to dribble and spatter … The actual shapes were largely determined by the dynamics of the material and his process: the viscosity of the paint, the speed and direction of its impact on the canvas … The result is so alive, so sensuously rich, that all earlier American painting looks pale by comparison." (Janson, p.846)

Nick Metropolis, Stan Ulam, and Johnny von Neumann at Los Alamos

Von Neumann to Gen. Richtmeyer, 1947:

I have been thinking a good deal about the possibility of using statistical methods to solve (nuclear devices) in accordance with the principle suggested by Stan Ulam. The more I think about it, the more I become convinced the idea has great merit.



What *is* the Monte Carlo Method?

- An actual bomb has some 100000000000000000000000000 neutrons flying around inside it.
- Traditionally, one would try to model $d(x,y,z,u,v,w,t)$, how many neutrons were at each point with each velocity at each time.
- Von Neumann said – let’s follow a small pollster’s sample of them – say 100! – using the ENIAC. Instead of keeping track of all the uranium nuclei, let’s just find the odds of each neutron hitting a nucleus at any given point, the odds of it splitting the nucleus, the odds of how many neutrons will come out and at what speeds and directions.
- Need to flip a *lot* of coins, so we get 100 pretend histories. Also must keep track of how the uranium heats up, how it explodes (photons), etc. etc. It’s a mini-simulation with dice. This is how the H-bomb was designed!

Randomness is cool

- Pollock found that spatter painting made a wonderfully energetic image, full of life. The school of abstract expressionism made this one of their favorite tools.
- Metropolis-Ulam-von Neumann found that throwing dice created realistic pseudo-worlds by which one can compute stuff in the real world. The Monte Carlo method is huge today in many types of calculations (like finding very large primes for banking encryption).
- <u>Is it a coincidence that both happened in the late-40's!?</u>

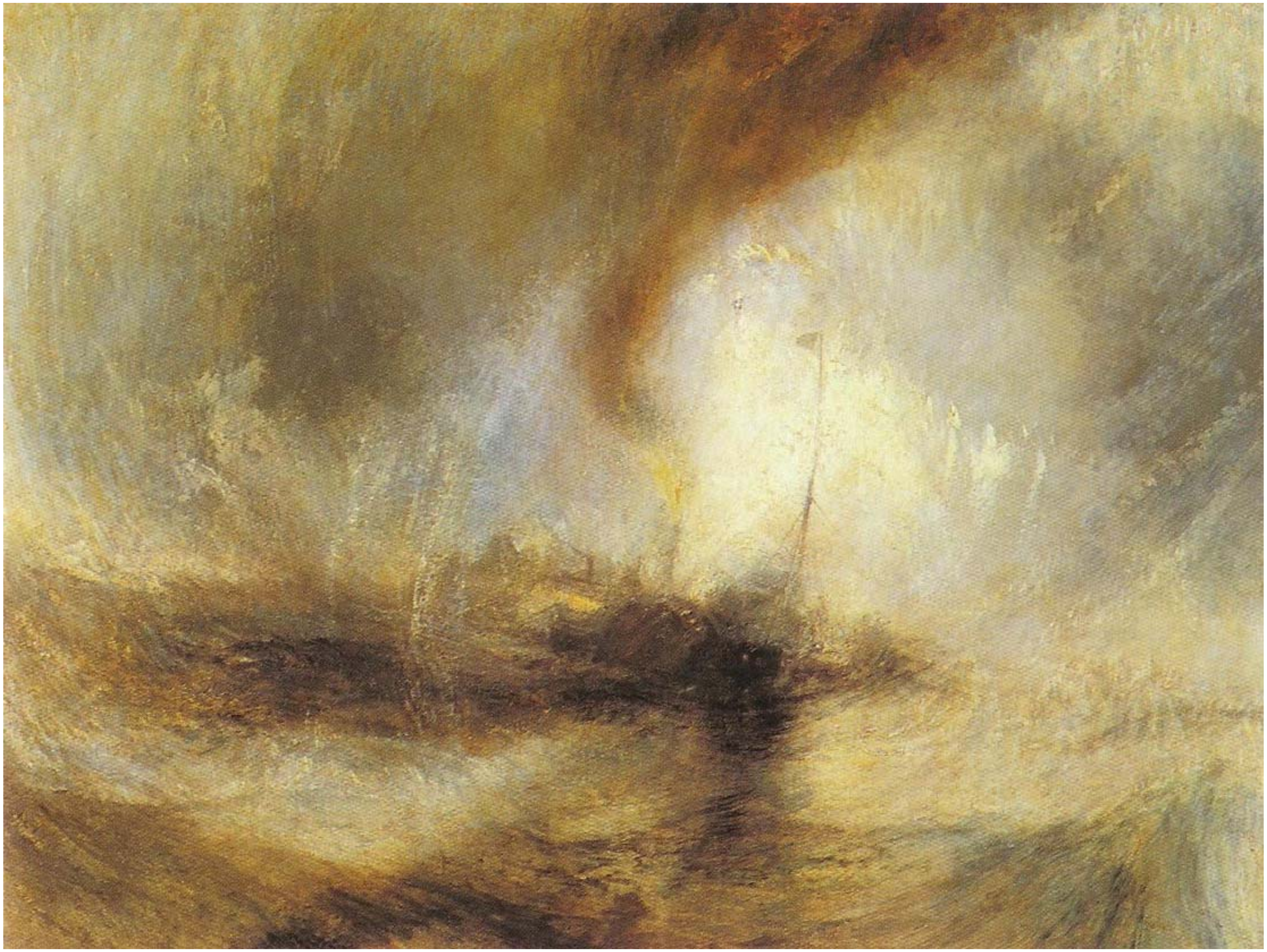
Let’s follow 200 years of history, during which abstract and non-figurative art developed

- Surely, you say, <u>all math is abstract and non-figurative</u>!
- **NO**: what is abstract depends on the perceiver. Dealing with *numbers as in Diophantus, geometry as in Euclid and processes in the world as in Newton* are the concrete ‘representational’ sides of math.
- Abstraction is a relative term: there are always ‘higher’ levels.
- The first stage historically: in the early 19th century, a focus on one aspect of a concrete situation, throwing out irrelevant details to get to the essence

Turner (1775-1851) ***Dido building Carthage,*** **1815**;
The age of romanticism: in love with golden light more than the objects themselves.

. **Turner's late work: *Steamer in a snowstorm*, 1842**

Objects dissolve and he paints pure light, water and air, mixed in mist, spray and clouds



Breaking the ties with the concrete in math:
Galois (1811-1832); Abel (1802-1829)

- 2 romantics whose ideas were rejected by the Academy, which could not understand what they were doing – it was too abstract.
- Galois died in a duel, Abel died of TB. Both were jobless and penniless, though their ideas were among the deepest of the 19th century.

The background: the ancient game of algebra

Challenges: "*Solve this – if you can*"

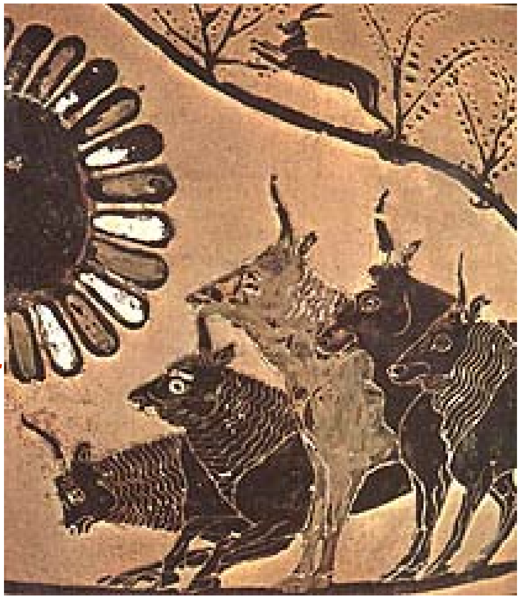
- Babylon (c.1800 BCE):

  "I have multiplied length and breadth and the area is 10. The excess of length over breadth I have multiplied by itself and this result by 9. And this area is the area obtained by multiplying length by itself. What are the length and breadth?" (A 4th degree equation)

- Archimedes (c. 250 BCE): challenges the mathematicians of Alexandria with the cattle problem (an equation whose simplest answer has 200,000 digits!)

To students of such matters at Alexandria, a letter from Archimedes to Eratosthenes of Cyrene. *If thou art diligent and wise, O stranger, compute the number of cattle of the Sun, who once upon a time grazed on the fields of the Thrinacian isle of Sicily, divided into four herds of different colours, one milk white, another a glossy black, a third yellow and the last dappled.*

…



Again, when the yellow and the dappled bulls were gathered into one herd they stood in such a manner that their number, beginning from one, grew slowly greater till it completed a triangular figure, there being no bulls of other colours in their midst nor none of them lacking.

If thou art able, O stranger, to find out all these things and gather them together in your mind, giving all the relations, thou shalt depart crowned with glory and knowing that thou hast been adjudged perfect in this species of wisdom.

The game continues in the Renaissance:

del Ferra, Tartaglia, Cardano and Ferrari (c.1500-1550). Mathematical contests raged in Venice and Milan:

“However, in the early hours of 13 February 1535, inspiration came to Tartaglia and he discovered the method ... Tartaglia was then able to solve all thirty of Fior's problems in less than two hours. As Fior had made little headway with Tartaglia's questions, it was obvious to all who was the winner.”

This is what ‘concrete, *tangible*’, algebra looked like in 1800

The quadratic formula you saw in school

$$x^2 + bx + c = 0$$

$$x = \frac{-b + \sqrt{b^2 - 4c}}{2}$$

The cubic formula of del Ferro:

$$x^3 + b.x^2 + c.x + d = 0$$

$$x = \sqrt[3]{-\frac{b^3}{27} + \frac{bc}{6} - \frac{d}{2} + \frac{1}{6}\sqrt{\frac{4c^3 - b^2c^2 + 4b^3d - 18bcd + 27d^2}{3}}} + \ldots$$

$$\sqrt[3]{-\frac{b^3}{27} + \frac{bc}{6} - \frac{d}{2} - \frac{1}{6}\sqrt{\frac{4c^3 - b^2c^2 + 4b^3d - 18bcd + 27d^2}{3}}}$$

What Galois did: he focused on one key aspect of the formulas, throwing out *all details*. Light and air alone

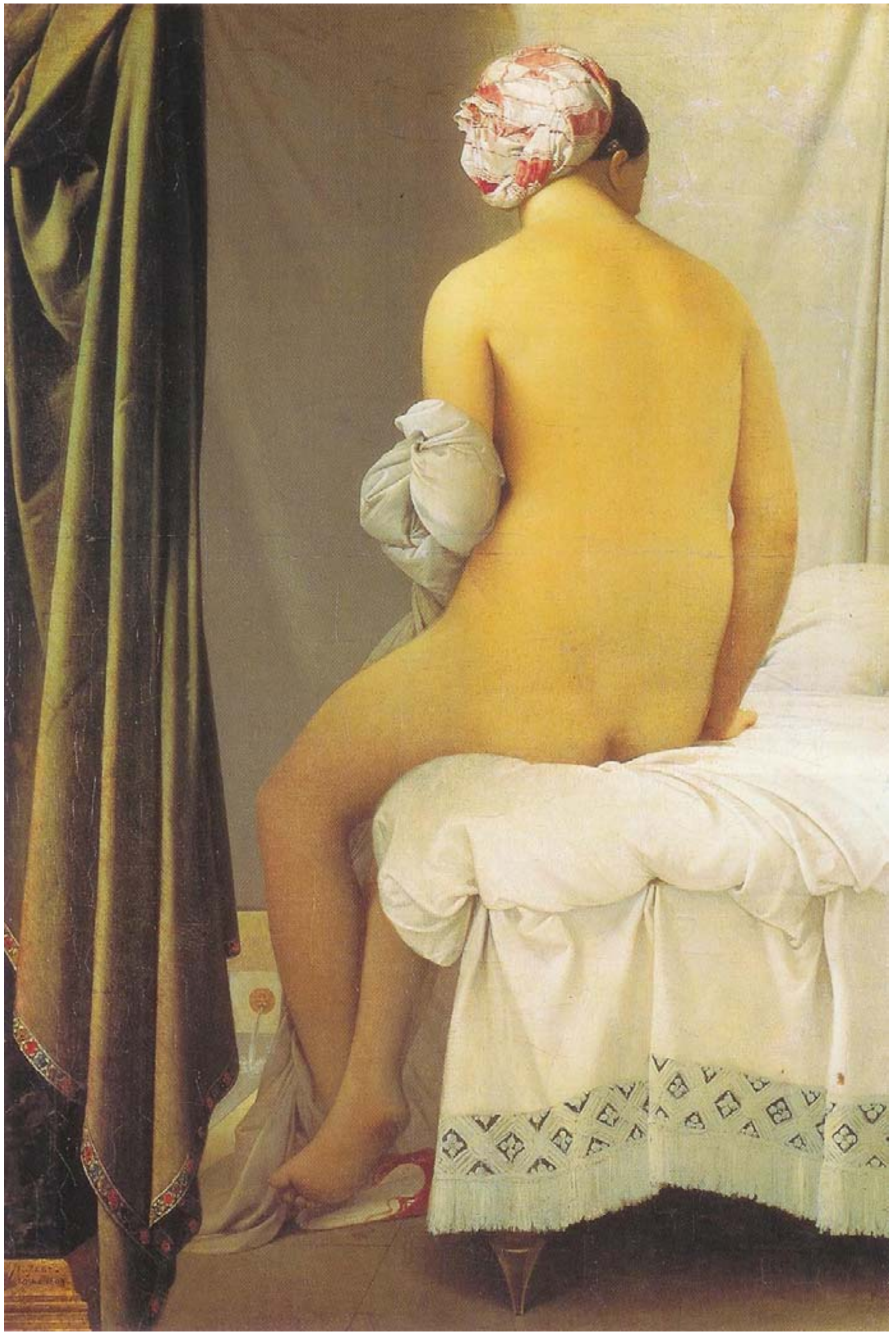
- He considered any possible formula for the solutions; if the degree is *n* there are *n* solutions.
- He rewrote all parts of the formula as expressions in these *n* solutions.
- He asked: what *rearrangements* of the solutions don't change these expressions. Example for the cubic:

$$\sqrt{\frac{4c^3 - b^2c^2 + 4b^3d - 18bcd + 27d^2}{3}} = (x_1 - x_2).(x_2 - x_3).(x_3 - x_1)$$

- He understood the formula as a way of step by step *decreasing* the number of these until there are none – and you have one solution by itself.

Phase II: the mid 19^{th} century

<u>Enlarging the aesthetic, nature in its richness offers beauty/depth in things classically deemed ugly.</u>

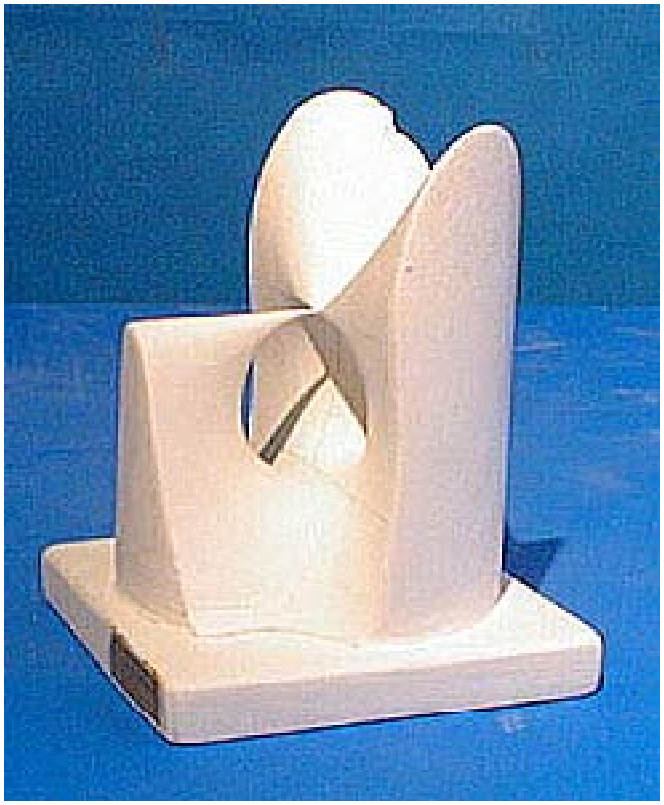


The classic ideal of perfection in art:

“Art should only depict beauty”, Ingres

The Valpincon Bather, 1808

‘Classical’ mathematics by the mid 19th century looked only at perfection too



Zeros of polynomials in 3 variables, real/imaginary parts of complex analytic functions – many were rendered in plaster by German craftsmen.

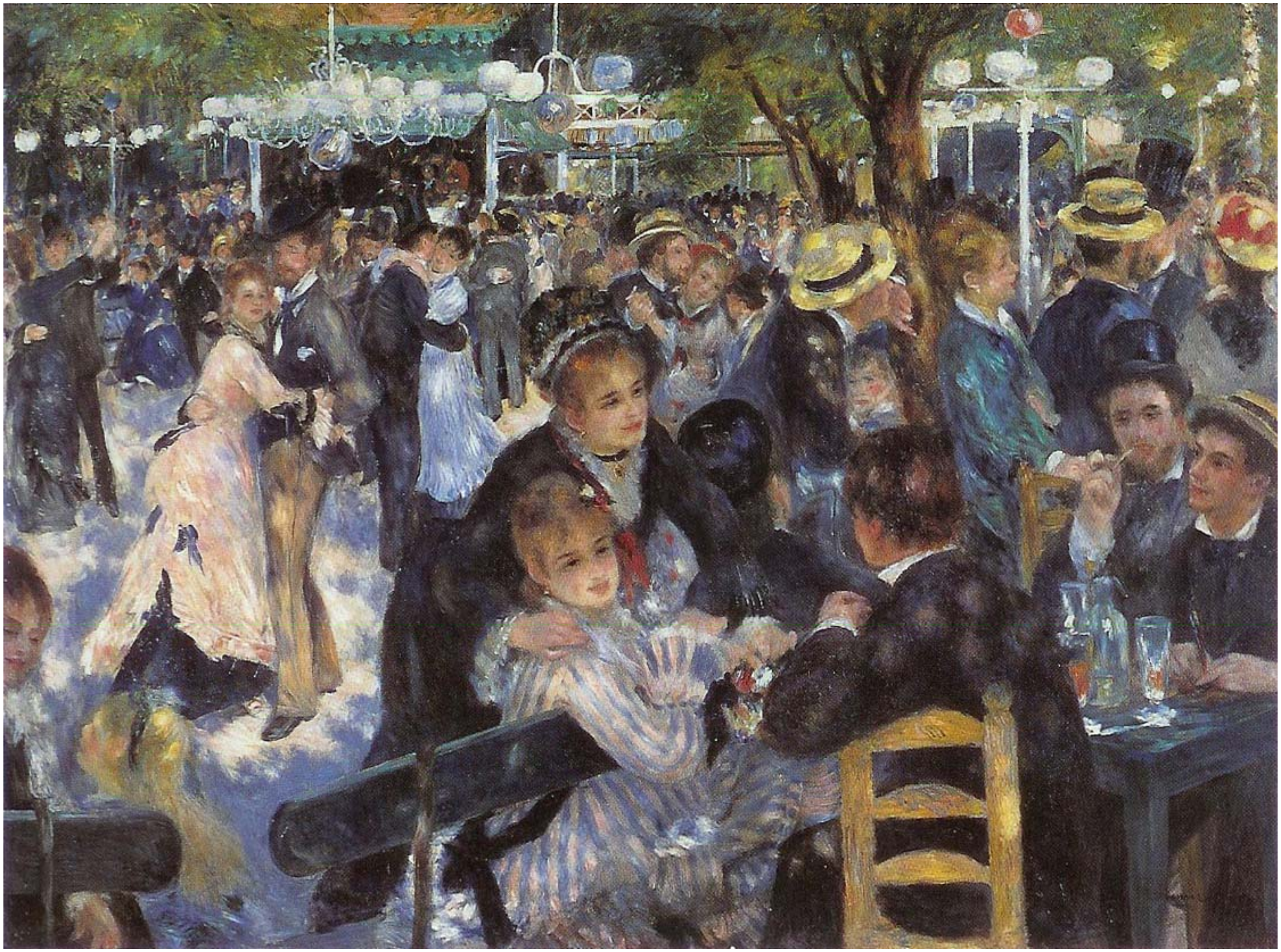
The wellspring of the most perfect ‘classical’ mathematics: $\sqrt{-1}$ = the imaginary unit

- Any fool knows all squares are positive, so what is this?
- Cardano after using $\sqrt{-1}$ *“So progresses arithmetic sublety, the end of which is as refined as it is useless”*
- Euler, the genius of the enlightenment, said

  “*…we are led to the idea of numbers which, from their nature, are impossible; and therefore they are usually called imaginary quantities, because they exist merely in our imagination. … not withstanding this, these numbers present themselves to the mind; they exist in our imagination and we still have a sufficient idea of them; … nothing prevents us from making use of these imaginary numbers, and employing them in calculation.”*
- And yet the most beautiful elegant mathematics ever discovered came from them.

Renoir, *Dance at the Moulin de la Galette*, 1876

Dappled shade forms a fractal pattern over faces, classical ideals of beauty are disregarded.



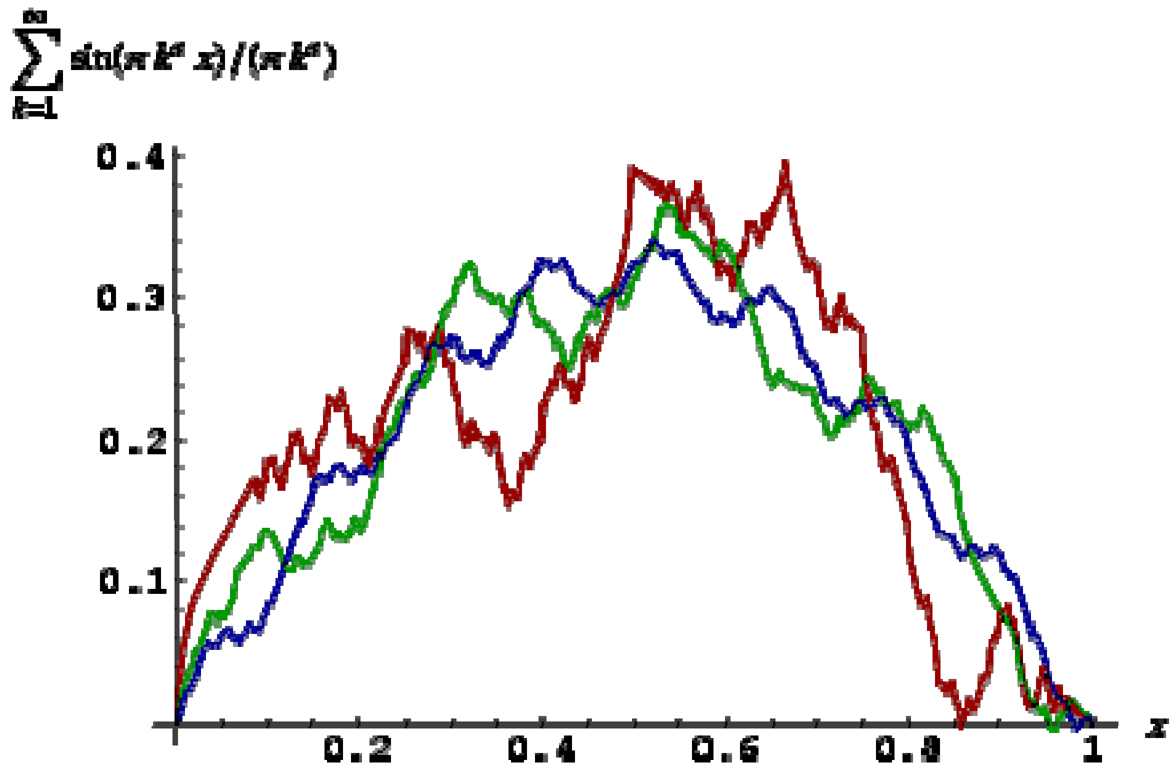
In math: Beautiful classical functions were fun – but to describe the non-smooth messy world, 'ugly' functions were needed

- Graph of Weierstrass's fcn.: no tangent lines anywhere

What is a 'function' anyway?

Was it a creation of God, a unique and important thing?

Or are functions simply a bag of all possible relationships, some well behaved, some wild; some smooth, some rocky; some beautiful, some ugly?

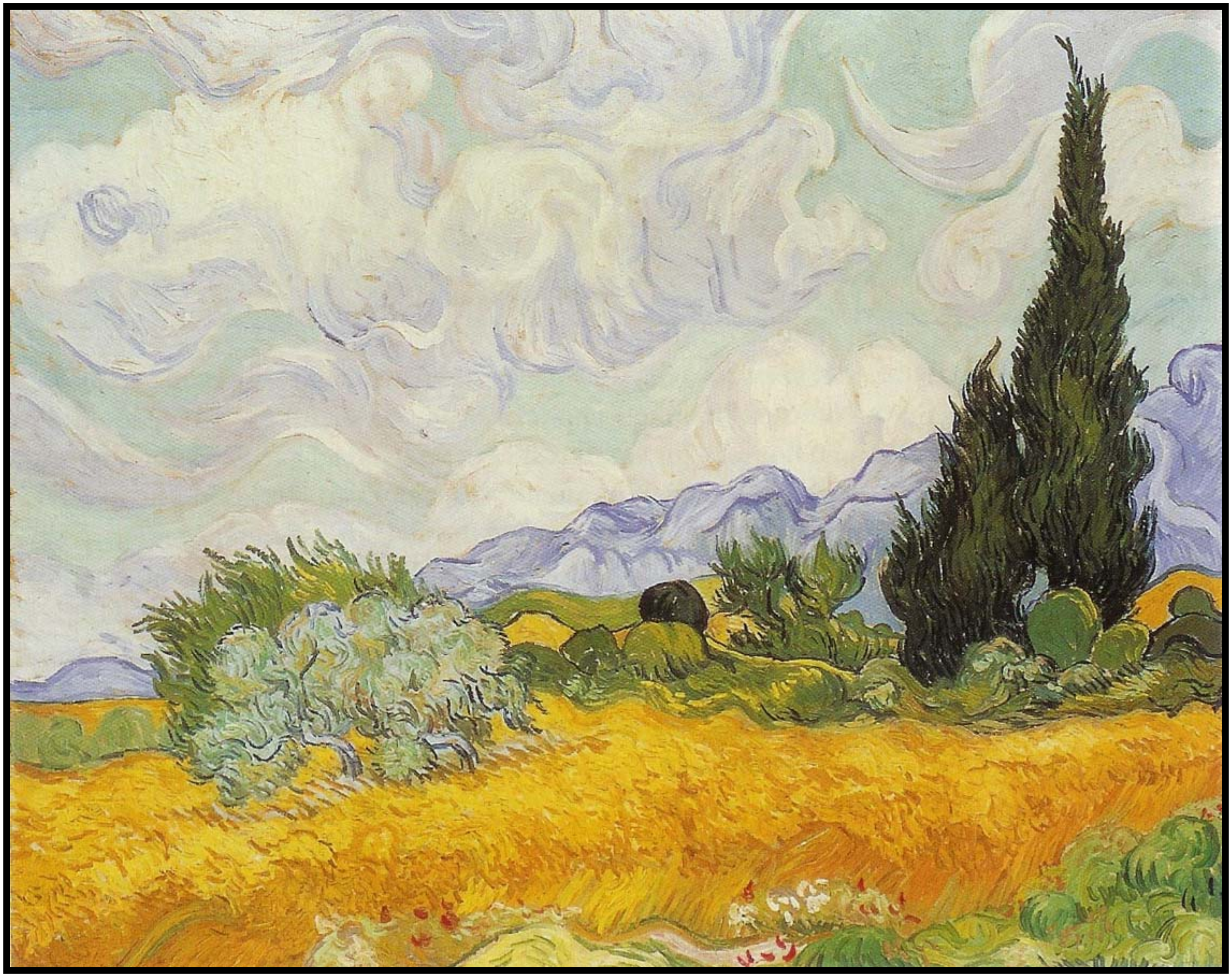


Phase III: The systematic creation of alternate and more vivid realities, counter-factual experiments with each part of our artistic/math world. In each, the real world is modeled differently, one or another element is omitted or changed.

Seurat, *The Bridge at Courbevoie*, 1886;
Form and texture are created out of dots, 'discrete geometry'

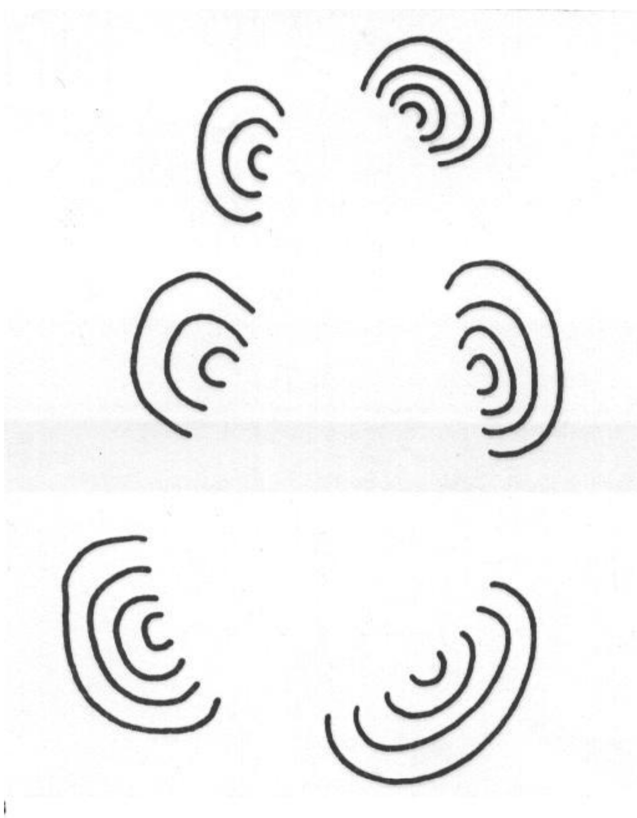
Van Gogh, *Cornfield with Cypresses,* 1889

Form and texture are created out of fluid swirls, ‘vector fields’



John Singer Sargent (1856-1925): Watercolors. The abstraction of light and shadow. You can omit details, the brain’s logic fills these in.

Digression: the scientific study of the <u>*grammar*</u> of perception (1890-1960)
The gestalt school, esp *Gaetano Kanisza*



Images in which the mind creates the implied shape

Wendy Artin: “Maurizio 3 seated from behind”, “Laura Sitting”

Mathematics explores different models of reality

- Felix Klein (1849-1925) *Erlangen Program* – different geometries defined by different groups of symmetries.
- Formalizing the axioms of logic: Gottlob Frege (1848-1925)
- Breaking down the constituents of geometry, algebra and analysis so that each theorem is set in ‘its natural generality’, i.e. in its own model universe.

Playing with the Elements of Math

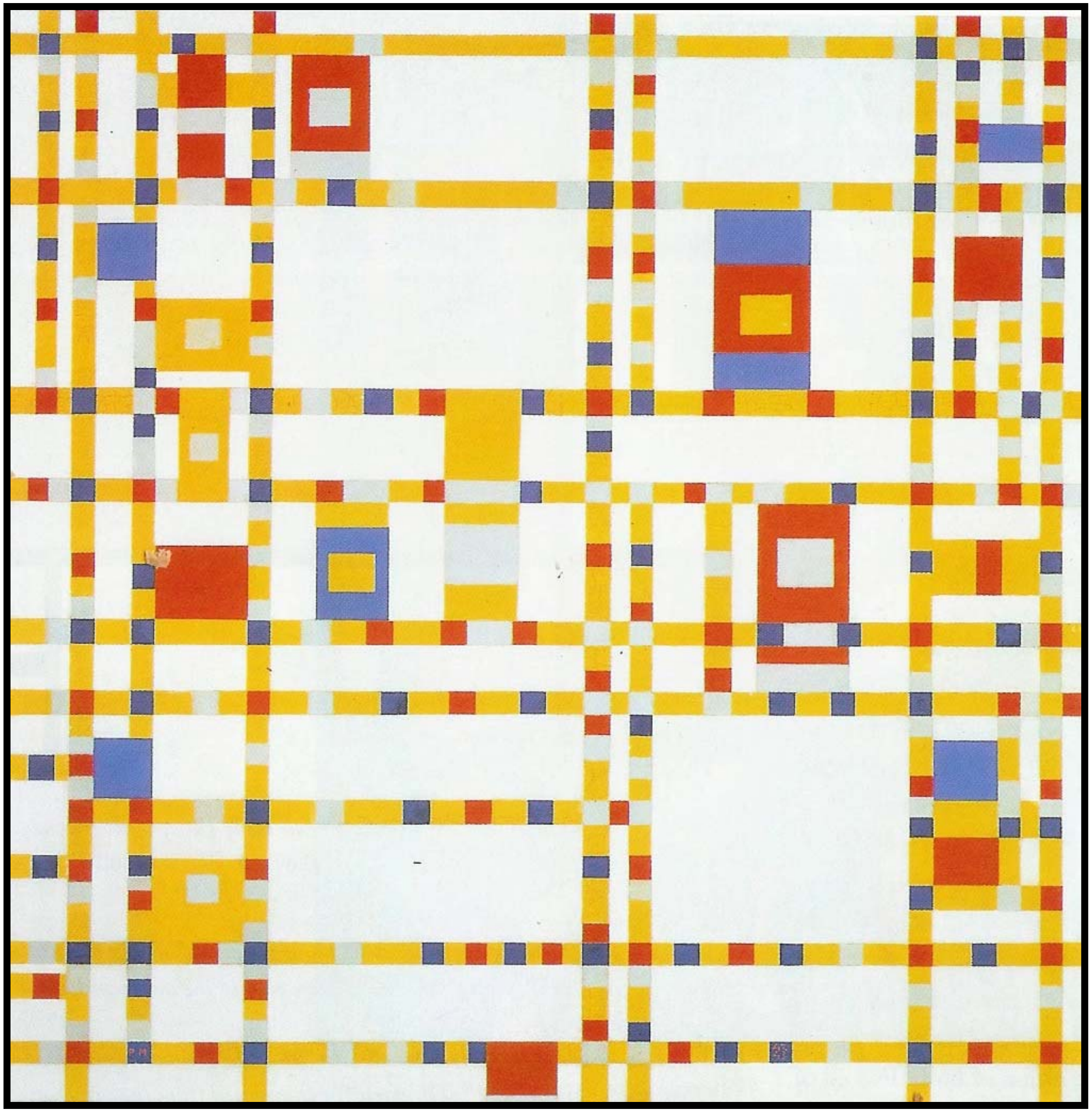
- Hilbert in *Grundlagen der Geometrie*, 1899, made the ultimate analysis of Euclid’s geometry, taking each axiom in turn and making an alternative geometry in which everything but that one axiom held. <u>A mind-game of partially real alternate models</u>.

A. 8 axioms of connection
(e.g. given 2 distinct pts, there is a unique line containing both)
B. 4 axioms of betweenness – none in Euclid!
(e.g. given 3 distinct pts on a line, exactly one is between the others)
C. 5 axioms of congruence
(e.g. 2 triangles with 2 sides and the included angle equal are congruent)
D. 1 parallel axiom – in a plane, let *l* be a line and *P* a pt off the line, then there is a unique line through the pt not meeting the line
E. 2 axioms of continuity:
Archimedes axiom: successive equal intervals cover the whole line
‘Eudoxian’ axiom: a sequence of nested intervals has a pt in the middle

Phase IV: Full blown abstraction

<u>Throw away all connection to conventional reality, the ‘reality’ of the painting/theory is not something *referred to* but something *constructed* by the art/math itself</u>

Kandinsky, *Cossacks*, 1910-11 (below) and Malevich:
Full blown non-figurative art



Mondrian,
***Broadway Boogie Woogie*, 1942**

There was a theory behind it

- “Art makes us realize that there are fixed laws which govern and point to the use of the constructive elements of the composition and the inherent inter-relationships between them.”
- “Non-figurative art is created by by establishing a dynamic rhythm of determinate mutual relations, which excludes the formation of any *particular* form.”

Mondrian, *Plastic Art and Pure Plastic Art,* 1937.

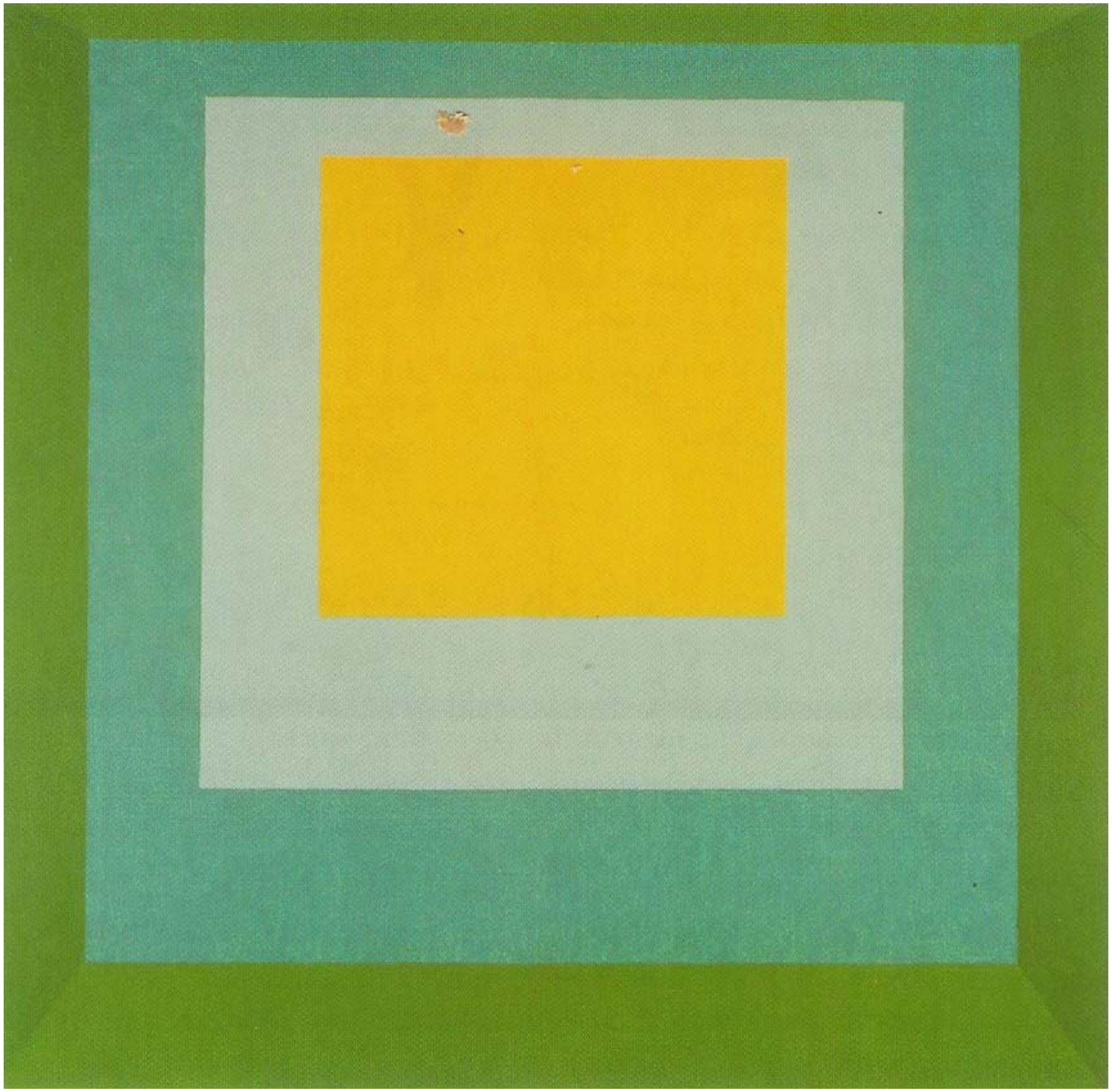
Math – at the same time – built everything up from *mutual relationships*: set theory

- Everything is a set, e.g. 5 is the <u>set of the five smaller numbers</u> 5={0,1,2,3,4}
- The natural numbers is the <u>infinite set</u> N={0,1,2,3,….}
- Addition is the <u>subset of NxNxN</u> of all triples {a,b,a+b}
- A positive fraction is a <u>maximal subset of NxN</u>, with any 2 pairs {a,b} and {c,d} in it satisfying a.d = b.c.
- The plane P is the <u>set of coordinate prs</u> {x,y}, i.e. RxR.
- A line L is a <u>subset of P</u> of pts in the plane satisfying ax+by+c=0.

- Set theory led to the modern-day incarnation of Euclid: Bourbaki’s *Elements de Mathematique* written by a French cooperative to give the supremely logical development of math
- It led to the infamous ‘new math’ which caused Generation X to think math was irrelevant.
- But it led to deep studies of *pure abstract algebra* – rings and groups, and the dissection of *the many types of structure that live in space* – simplicial, Riemannian, complex.

Rothko, *White and Greens in Blue*, 1957;

"Modern art…actively engages with the myriad ideas in the contemporary environment…In an age that is pre-occcupied with the dissection of matter to arrive at the basis of its structural life, where all perceptible phenomena are being dissolved into their abstract components, art can do nothing else but follow the same course in relation to the laws of art." *Mark Rothko*, 1940's.

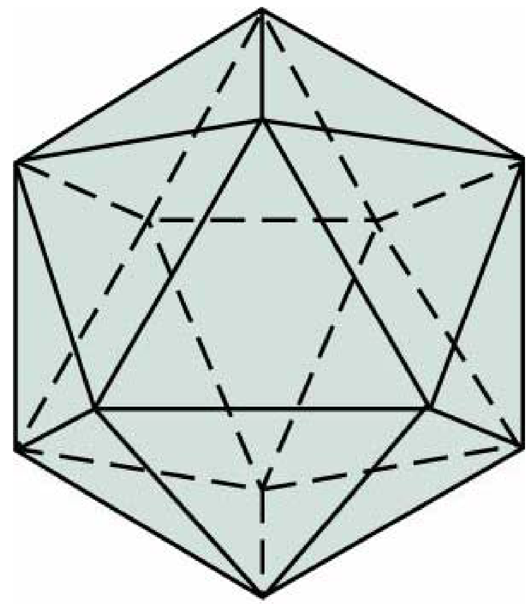


Albers, *Apparition, Homage to the Square*,1959

“(The) choice of the colors used, as well as their order is aimed at an interaction …. Though the underlying and quasi-concentric order of squares remains the same in all paintings, these same squares group or single themselves, connect and separate in many different ways”, *On my “Homage to the Square”* (1964)

Minimalism: reduction to the simplest conceivable structures

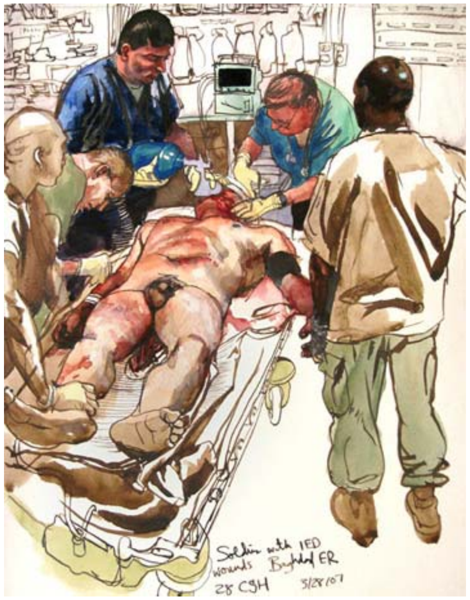
- In Art, minimalism appears in the careers of Malevich, Albers, Kelly, Stella, LeWitt, Noland.
- In Math, we have the classification of ‘finite, simple groups’ (1955-1983)
- These are the most basic finite sets of elements, any 2 of which can be composed to get a 3rd. <u>Like the smile of the Cheshire cat, they are the symmetry when the symmetric object is taken away.</u>



Academy Artworks

There are 60 rotations which ‘preserve’ the icosahedron. These 60 make up the simplest non-trivial ‘finite simple group’.

Post-modernism? Well, here’s a sample from my family. Eclectic, every technique is in play.



Stephen Mumford, Andrew Moore, Inka Essenhigh, Peter Mumford, Jenifer Mumford

The same is happening in math: eclectic, every technique is in play.

- Partial differential equations – which originated to model waves – were used by Hamilton and Perelman to solve the Poincaré conjecture in pure topology
- Modular forms (from analysis) were used by Wiles to solve Fermat’s conjecture in pure number theory.

Why are the parallels so tight?

- It seems to me, as a professional in Math and a lover of Art, that for 200 years there has been an *uncanny* similarity between each new approach in Art and in Math.
- Rarely is there any direct influence, e.g. A wondering if B's ideas could be carried over from Math to Art or Art to Math. (Duchamp and Einstein? Rothko and Los Alamos?)
- But maybe we should accept that we are merely pawns, while the real player is the *zeitgeist*!