Pseudo-code to maximize

$$p(\{\ell_\alpha\}|\vec{\theta}_\ell) = \frac{1}{Z} e^{-(\lambda \sum_{\alpha\sim\beta} \mathbb{1}(\ell_\alpha \neq \ell_\beta) + \mu \sum_\alpha E_\alpha(\ell_\alpha|\vec{\theta}_{\ell_\alpha}))}$$

$$B(\vec{e},\ell) = \left(\begin{array}{c} \text{belief in label } \ell \text{ at vertex } \beta \text{ conveyed} \\ \text{by directed edge } \vec{e}:\alpha \to \beta \end{array} \right)$$

Iterate:

A) E-step. Fix $\{\theta_\ell\}$ at last value.

Initialize $B(\vec{e},\ell) = 1/L$

Iterate

$$B(\vec{e},\ell) = \frac{1}{Z} \sum_{\ell'=1}^{L} e^{-\lambda \mathbb{1}(\ell \neq \ell')} \cdot e^{-\mu E_\alpha(\ell')} \cdot \prod_{f_1,f_2,f_3} B(\vec{f}_i,\ell')$$

where

$\vec{e}:\alpha \to \beta$ is any directed edge

$\vec{f}_i:\gamma_i \to \alpha$ are the 3 edges connecting to $\vec{e}$

$Z =$ constant so that $\sum_\ell B(\vec{e},\ell) = 1$

Output approx. probabilities

$$\text{probs}\ (\alpha,\ell) = \frac{1}{Z} e^{-\mu E_\alpha(\ell)} \prod_{f_1,f_2,f_3,f_4} B(\vec{f}_i,\ell)$$

where

$\vec{f}_i:\gamma_i \to \alpha$ are the 4 edges ending at α

$Z =$ constant so that $\sum_\ell$ probs $(\alpha,\ell) = 1.$

B) M-step: Update $\{\theta_\ell\}$

For all ℓ maximize

$$\sum_\alpha \text{probs}\ (\alpha,\ell) E_\alpha(\ell|\vec{\theta}_\ell)$$

with respect to $\vec{\theta}_\ell$.