AM18, Assignment #3, due Monday Feb. 13

The file `balltoss.jpg` is a composite photograph of a squash ball being thrown up in front of my blackboard (created from a digital camera set in movie mode).

1. First bring it up in a program that allows you to track the cursor position (e.g. Microsoft Photo Editor or ImageJ – as suggested by Matt). Then you should measure the 15 positions (x_i, y_i) of the ball in its arc. Note that because of the exposure time, each shot shows a blurry streak rather than a ball. I suggest you measure both the initial and final points of the streak as accurately as possible. This way, you get a good measure of the velocity as well as the position at all 15 times. Also, measure the outer corners of the blackboard. Since, in the real world, it is 4 feet by 8 feet, you can use this to convert the measurements of the ball position on the photo into feet. (I know there is also perspective distortion, but ignore this.) The frame rate for the movie was 15 frames per second, so we have calibrated time measurements too.

2. We want to analyze how well these conform to the equations of motion given by Galileo. So next put these numbers in columns in Excel (e.g the time in column A, starting at 0 and adding 1/15 second for each observation, and then your measurements in the next columns).

3. Next look at velocities. Compute the horizontal and the vertical velocities as a function of time. You can either use differences between the first and last point of each streak made by the squash ball or the difference between the first points of consecutive streaks. Plot both the horizontal and vertical velocity – call them u and v – against time and print this out. Fit a straight line to each (drawn by hand unless you want to be fancy and sue Excel for this too): is the horizontal velocity roughly constant as predicted or nor? Don't expect any of the laws to be followed exactly: there is air resistance as well as projective distortion (i.e. the camera viewing geometry). Find the time t_0 when the vertical velocity is zero by the zero crossing of the fitted line (this will probably not be exactly at any single observation but instead be between them). Measuring the slope of the line fitted to vertical velocity, estimate the vertical acceleration downwards (which we call g).

4. Then look at positions. Fit Galileo's law as follows. For horizontal motion, take the average $\bar{u}$ of the horizontal velocities found above, and fit the horizontal position with $x \approx \bar{u}t + x_0$. Then graph the predicted x as well as the measured x and see how close they come to each other. Print out this graph. For vertical positions, we have estimated in part 3 the high point t_0 of the trajectory as well as the acceleration downwards g. So the predicted vertical position should be $-g(t-t_0)^2/2 + y_0$ where y_0 is the position of the ball at the apex of its trajectory. Estimate y_0, plot the predicted y and the measured y against time and print this out. Finally make a plot of the predicted x and y (no time in this plot) and the measured x and y from the photo and see whether the shape of the trajectory has come out right!