PDE, HW 4, due Monday 10/31/05

1. Problem 2, John, p. 213

2. Problem 3, John, p. 213.

3. Problem 7, John p. 213

4. Problem 12, John p. 214

5. Problem, John p. 220

6. Construct solutions of the heat equation that are defined for all $t \in \mathbb{R}$.

7. Let f be an integrable function in $\mathbb{R}^n$ with $\int_{\mathbb{R}^n} f = 1$. Observe that if we rescale $f_\lambda(x) := \lambda^n f(x\lambda)$ then $\int_{\mathbb{R}^n} f_\lambda = 1$. Moreover as $\lambda \to \infty$, for any bounded continuous function φ we have $\lim_{\lambda\to\infty} \int_{\mathbb{R}^n} f_\lambda(x)\varphi(x) \to \varphi(0)$. Therefore, f_λ behaves like δ_0 for large λ.

Define the solution to the heat equation $u(x,t) = \int_{\mathbb{R}^n} k(x-y,t)f(y)dy$ with initial data f. The behavior of initial data under rescaling, and the self-similarity of $k(x,t)$ suggest that asymptotically the solution $u(x,t)$ should not be very different from $k(x,t)$. Formulate and prove a precise version of this.