

# Linear Algebra

MA 242 (Spring 2013)

Instructor: M. Chirilus-Bruckner

## DETERMINANT

–  $4 \times 4$  example –

Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 0 & 4 \\ 3 & 2 & 0 & -1 \\ 0 & 2 & -3 & 1 \end{bmatrix}.$$

**Solution:** Cofactor expansion along **column 3**.

$$\det \begin{bmatrix} 1_{+} & 0_{-} & 2_{+} & 3 \\ 2 & -1 & 0_{-} & 4 \\ 3 & 2 & 0_{+} & -1 \\ 0 & 2 & -3_{-} & 1 \end{bmatrix}$$

$$= 2(-1)^{1+3} \det \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$+ 0(-1)^{2+3} \det \begin{bmatrix} 1 & 0 & 3 \\ 3 & 2 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$+ 0(-1)^{3+3} \det \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ 0 & 2 & 1 \end{bmatrix}$$

$$+ (-3)(-1)^{4+3} \det \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ 3 & 2 & -1 \end{bmatrix}$$

$$= 2(4 + 0 + 24 - 0 - (-3) - (-4)) \quad (\text{by rule of Sarrus})$$

$$= -3(-1)(1(1 - 8) + 3(4 + 3)) \quad (\text{by cofactor expansion})$$

$$= 112$$