Millersville University
Department of Mathematics
MATH 161, *Calculus I*, Test 4
December 5, 2006

Name Answer Key

Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (8 points each) Evaluate the following definite and indefinite integrals exactly (no numerical approximations).

(a) $\int \frac{x}{x^2+9}\,dx$

Integrate by substitution:

$$u = x^2+9$$

$$\frac{1}{2}\,du = x\,dx$$

$$\int \frac{x}{x^2+9}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(x^2+9) + C$$

(b) $\int_{-1}^{3}(x^3-2x)\,dx$

$$= \frac{1}{4}x^4 - x^2\Big|_{-1}^{3}$$

$$= \frac{81}{4} - 9 - \left(\frac{1}{4} - 1\right)$$

$$= \frac{81-1}{4} - 9 + 1$$

$$= 20 - 8$$

$$= 12$$

(c) $\int e^x(2+e^x)^2\,dx$

Integrate by substitution:

$u = 2+e^x$

$du = e^x\,dx$

$$\int e^x(2+e^x)^2\,dx = \int u^2\,du$$
$$= \frac{1}{3}u^3 + C$$
$$= \frac{1}{3}(2+e^x)^3 + C$$

(d) $\int_0^2 xe^{-x^2}\,dx$

Integrate by substitution:

$u = -x^2$

$-\frac{1}{2}\,du = x\,dx$

$$\int_0^2 xe^{-x^2}\,dx = -\frac{1}{2}\int_0^{-4} e^u\,du$$
$$= -\frac{1}{2}e^u\Big|_0^{-4}$$
$$= -\frac{1}{2}e^{-4} + \frac{1}{2} \approx 0.490822$$

(e) $\int 3x\sec(2x^2)\tan(2x^2)\,dx$

Integrate by substitution:

$u = 2x^2$

$du = 4x\,dx$

$\frac{3}{4}\,du = 3x\,dx$

$$\int 3x\sec(2x^2)\tan(2x^2)\,dx = \int \frac{3}{4}\sec u\tan u\,du$$
$$= \frac{3}{4}\sec u + C$$
$$= \frac{3}{4}\sec(2x^2) + C$$

(f) $\int_1^e \frac{\ln x}{x}\, dx$

Integrate by substitution:

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$\int_1^e \frac{\ln x}{x}\, dx = \int_0^1 u\, du$$

$$= \frac{1}{2} u^2 \Big|_0^1$$

$$= \frac{1}{2}$$

(g) $\int \frac{x^3}{\sqrt{1-5x^4}}\, dx$

Integrate by substitution:

$u = 1 - 5x^4$

$du = -20x^3\, dx$

$\frac{-1}{20}\, du = x^3\, dx$

$$\int \frac{x^3}{\sqrt{1-5x^4}}\, dx = \frac{-1}{20} \int \frac{1}{\sqrt{u}}\, du$$

$$= \frac{-1}{20} \int u^{-1/2}\, du$$

$$= \frac{-1}{20} \cdot 2\, u^{1/2} + C = \frac{-1}{10}\sqrt{1-5x^4} + C$$

2. (8 points) Use summation rules to evaluate the sum:

$$\sum_{i=1}^{123} (i^2 + 2i + 1).$$

$$\sum_{i=1}^{123} (i^2+2i+1) = \sum_{i=1}^{123} i^2 + 2\sum_{i=1}^{123} i + \sum_{i=1}^{123} 1$$

$$= \frac{123(123+1)(2(123)+1)}{6} + 2\left(\frac{123(123+1)}{2}\right) + 123$$

$$= \frac{(123)(124)(247)}{6} + (123)(124) + 123$$

$$= 627874 + 15252 + 123 = 643249$$

3. (10 points) Use a Riemann sum with $n = 12$ and midpoint evaluation points to approximate the area under the graph of $f(x) = \sqrt{2x+1}$ on the interval $[0,3]$.

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{12} = 1/4$$

$$c_i = a + \frac{(2i-1)}{2}\Delta x = \frac{2i-1}{8}$$

$$\text{Area} \approx \sum_{i=1}^{12} \sqrt{\frac{2i-1}{4} + 1}\ (1/4) \approx 5.84169$$

4. (8 points) Find the derivative of $f(x)$ where

$$f(x) = \int_{-1}^{\tan x} \sqrt{t^2+1}\, dt.$$

By the FTC, Part II:

$$f'(x) = \sqrt{\tan^2 x + 1} \cdot \frac{d}{dx}(\tan x)$$

$$= \sqrt{\sec^2 x} \cdot \sec^2 x$$

$$= \sec^3 x$$

5. (8 points) Find the average value of the function $f(x) = 2x - x^3$ on the interval $[0,3]$.

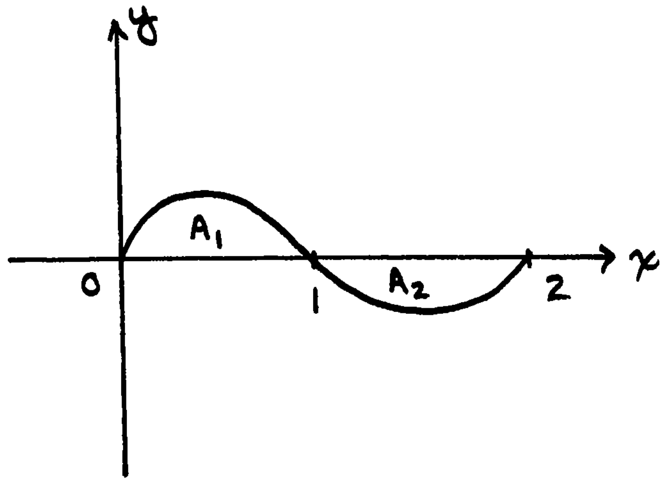
$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\, dx$$

$$= \frac{1}{3-0}\int_0^3 (2x - x^3)\, dx$$

$$= \frac{1}{3}\left(x^2 - \frac{1}{4}x^4\right)\Big|_0^3$$

$$= \frac{1}{3}\left(9 - \frac{81}{4}\right) = -\frac{15}{4} = -3.75$$

6. (10 points) Express the area between the graph of $y = x^3 - 3x^2 + 2x$ and the x-axis on the interval $[0, 2]$ as a sum or difference of definite integrals.



$$\text{Area (total)} = \int_0^1 (x^3 - 3x^2 + 2x)\,dx - \int_1^2 (x^3 - 3x^2 + 2x)\,dx$$

$$= \frac{1}{4}x^4 - x^3 + x^2 \Big|_0^1 - \left[\frac{1}{4}x^4 - x^3 + x^2 \Big|_1^2 \right]$$

$$= \frac{1}{4} - 1 + 1 - \left[(4 - 8 + 4) - (1/4 - 1 + 1) \right]$$

$$= \frac{1}{4} - (-1/4)$$

$$= 1/2$$