Chapter 3 Review

1 Probelms on Implicit Differentiation

#1. A function is defined implicitly by

$$x^3y - 3xy^3 = 3x + 4y + 5.$$

Find y' in terms of x and y.

In problems 2-6, find the equation of the tangent line to the curve at the given point.

#2. $\dfrac{x^3+1}{y} + 2y^2 = 1 - 2x + 4y$ at the point $(2, -1)$.

#3. $4e^y + 3x = \dfrac{1}{x} + (y+1)^2 + 5x$ at the point $(1, 0)$.

#4. $(3x - 2y)^2 + x^3 = y^3 - 2x - 4$ at the point $(1, 2)$.

#5. $\sqrt{xy} + x^3 = y^{3/2} - y - x$ at the point $(1, 4)$.

#6. $x\sin(y-3) + 2y = 4x^3 + \dfrac{2}{x}$ at the point $(1, 3)$.

#7. Find y'' for the curve

$$xy + 2y^3 = x^3 - 22y \text{ at the point } (3, 1).$$

#8. Find the points at which the curve $x^3y^3 = x + y$ has a horizontal tangent.

2 Problems on Local Linearization

#1. Let $f(x) = (x+2)e^x$. Find the value of $f(0)$. Use this to approximate $f(-.2)$.

#2. $f(2) = 4$ and $f'(2) = 7$. Use linear approximation to approximate $f(2.03)$.

#3. $f(1) = 9$ and $f'(x) = \dfrac{6x^4}{x^2+1}$. Use a linear approximation to approximate $f(1.02)$.

#4. A linear approximation is used to approximate $y = f(x)$ at the point $(3, 1)$. When $\Delta x = .06$ and $\Delta y = .72$. Find the equation of the tangent line.

3 Problems on Absolute Maxima and Minima

#1. For the function $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 1$, find the x-coordinates of the absolute max and absolute min on the interval

- a) $-3 \leq x \leq 5$
- b) $0 \leq x \leq 5$

#2. Find the global max and the global min of $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3} + 2$ on the interval $-1 \leq x \leq 4$.

#3. Find the global max and global min of $f(x) = \frac{16}{3x^3} - \frac{3}{x^2} - \frac{1}{x}$ on the interval $1 \leq x \leq 4$.

#4. Find the global max and global min of $f(x) = \frac{1}{2}x + \frac{1}{2}\cos 2x$ on the interval $0 \leq x \leq 2\pi$.

4 Problems on Increasing and Decreasing Functions

In each problem, find the intervals on which the function is increasing and the intervals on which the function is decreasing. Use this information to find the x-coordinates of the local maxima and local minima. Check your results by using your calculator to produce a sketch of the graph.

#1. $f(x) = 2x^3 + 15x^2 - 144x + 16$.

#2. $\dfrac{8 - x - x^2}{x^3}$.

- Hint: To make the differentiation easier, try dividing the top by the bottom first.

#3. $\frac{3}{40}(x-5)^{2/3}(5x^2 + 14x + 105)$.

- Hint: To make the differentiation easier, try multiplying out the x-stuff first.

5 Problems on Concavity

For the functions below, determine the intervals on which the graph is concave up and intervals on which the graph is concave down. Find the x-coordinates of any points of inflection. Then, use your calculator to draw a graph of the function to check your results.

#1. $f(x)\frac{1}{60}(3x^5 + 20x^4)$.

#2. $f(x) = \frac{9}{10}x^{5/3} + 36x^{2/3}$.

#3. $f(x) = f(x) = \dfrac{6x^2 - 16x + 15}{12x^3}$.

#4. $f(x) = \frac{9}{140}(2x^{10/3} - 25x^{7/3})$.

#5. $f(x) = \dfrac{x}{x^2 - 4}$.

6 Max-Min Word Problems

#1. A rectangular box with a square bottom and no top is to have a volume of 160 cubic feet. The material for the bottom costs $5 per square foot, while the material for the sides costs $8 per square foot. Find the dimensions of the box having the smallest possible cost.

#2. A piece of string 2 feet long is divided into two pieces. One piece is used to make a whole circle, while the other is used to make an equilateral triangle. How should the string be divided in order to make the sum of the areas of the circle and triangle smallest.

#3. Find the points on $y = x^2$ which are closest to $(0, 12)$.

#4. A ten-foot pole and a twenty-foot pole stand 75 feet apart. A wire is to run from the top of one pole to ground between them and then to the top of the second pole. How far from the 10-foot pole should the wire reach the ground in order to minimize the total length of the wire.

#5. Find the positive number for which the sum of twice the number and times its reciprocal is smallest.

#6. A field consists of a rectangular section with two semicircular sections attached to opposite sides. If the perimeter of the field is to 400 yards, what dimensions give the field of largest area?

7 Problems on Related Rates

#1. x and y are related by the equation

$$(2x + y)^{10} + 3x = 9 + 5y^3.$$

Find the rate at which y is changing when $x = 1$ and $y = -1$, if x decreases at 2 units per second.

#2. A rectangle has width x and height y. Find the rate at which x is changing when $x = 12$ and $y = 14$, if y is increasing at 5 units per second and the area is decreasing at 2 units per second.

#3. Two long parallel paths are 10 feet apart. You stand on one path holding a leash attached to toddler. The toddler runs along the other path in the direction away from you at 3 feet per second. Assuming that you let out the leash so that the leash remains taut, at what rate is the acute angle between the leash and your path changing when you and the toddler are 26 feet apart.

#4. A six foot tall Philly Phanatic walks away from a 15 foot lamp post at a constant rate of 2 feet per second. Let θ be the angle formed by the top of the lamp post, the tip of the Phanatic's shadow, and the foot of the lamp post. At what rate is θ changing when the Phanatic is 6 feet from the lamp post?