Millersville University
Department of Mathematics
MATH 161, *Calculus I*, Test 3
November 10, 2006

Name Answer Key

Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (4 points each) Consider the function

$$f(x) = x^3 - \frac{3}{400}x.$$

(a) Find the horizontal and vertical asymptotes of the function, if any.

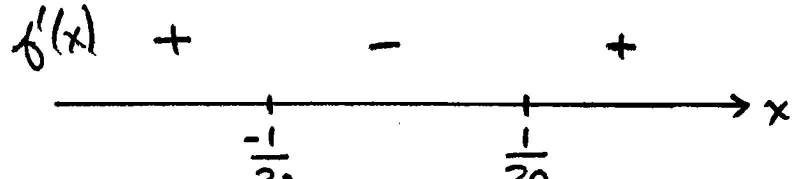
$f(x)$ is a polynomial and thus has no horizontal or vertical asymptotes.

(b) Find the critical numbers of the function (if any) and the intervals where the function is increasing and decreasing.

$f'(x) = 3x^2 - \frac{3}{400}$ and $0 = 3x^2 - \frac{3}{400}$

$x^2 = \frac{1}{400}$

thus $x = \pm\frac{1}{20}$ are the only critical numbers.



$f(x)$ is increasing on $(-\infty, -1/20) \cup (1/20, \infty)$ and decreasing $(\frac{-1}{20}, \frac{1}{20})$.

(c) Find the local extrema of the function, if any. Classify each extremum as a local minimum or local maximum.

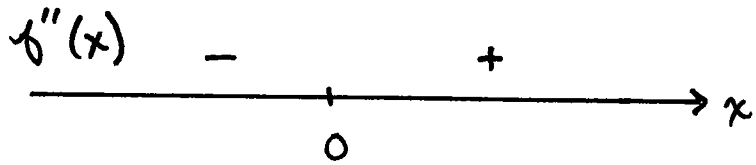
By the First Derivative Test,

$f(-1/20) = \frac{1}{4000}$ is a local maximum, and

$f(1/20) = \frac{-1}{4000}$ is a local minimum.

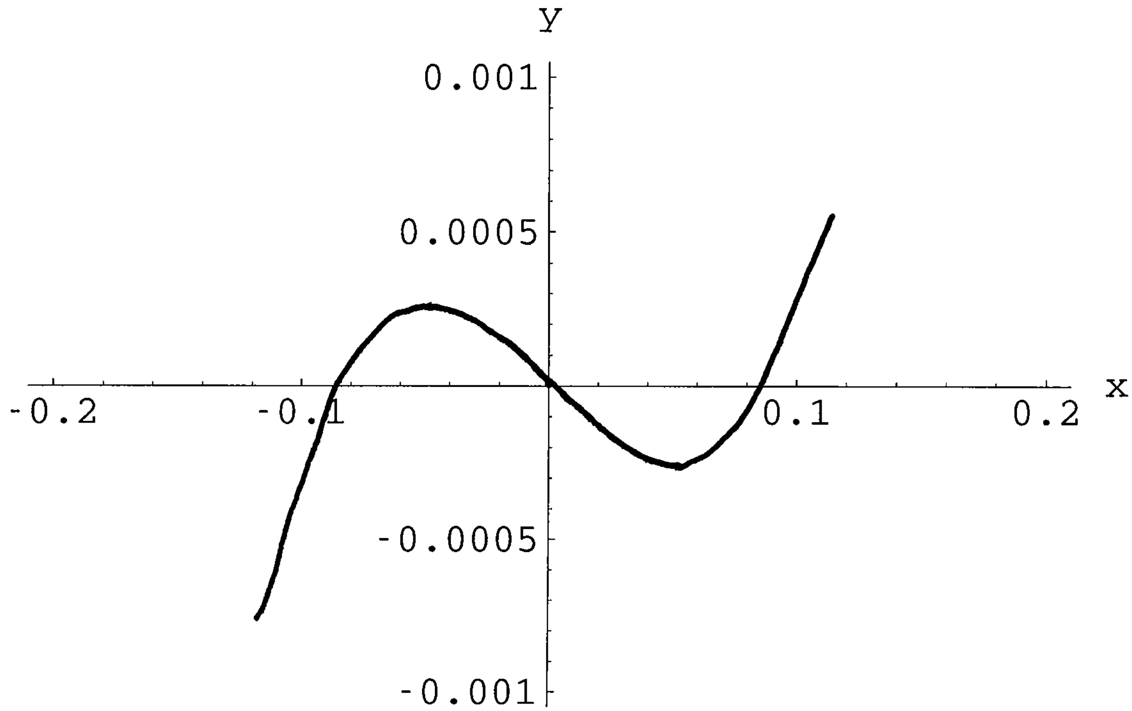
(d) Find the points of inflection of the function (if any) and the intervals where the graph of the function is concave up and concave down.

$f''(x) = 6x$, if $6x = 0$, then $x = 0$.

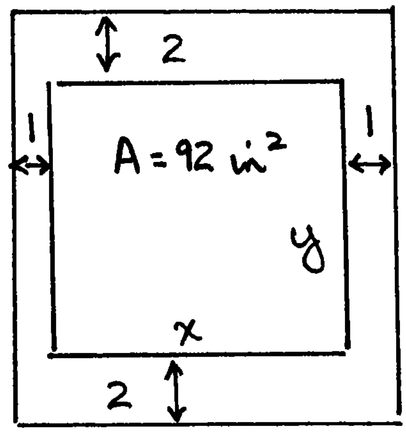


$f(x)$ is concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$.

(e) Sketch a graph of the function on the axes provided below.



2. (10 points) An advertisement consists of a rectangular printed region plus 1-inch margins on the sides and 2-inch margins on the top and bottom. The area of the printed region is to be 92 inch^2. Find the dimensions of the printed region that minimize the total area of the advertisement.



$92 = xy$

$\Rightarrow \; y = 92/x$.

Area of advertisement, $A = (x+2)(y+4)$

$$= (x+2)\left(\frac{92}{x}+4\right)$$

$$\frac{dA}{dx} = \left(\frac{92}{x}+4\right) + (x+2)\left(\frac{-92}{x^2}\right)$$

$$= \frac{92}{x} + 4 - \frac{92}{x} - \frac{184}{x^2}$$

$$\frac{dA}{dx} = 4 - \frac{184}{x^2}$$

critical numbers: $0 = 4 - \frac{184}{x^2}$

$x^2 = 46$

$x = \sqrt{46} \approx 6.78$

$\frac{d^2A}{dx^2} = \frac{368}{x^3} > 0$ when $x = \sqrt{46}$. By the Second Derivative Test, a local minimum occurs when

$x = \sqrt{46} \approx 6.78$ in.

$y = 2\sqrt{46} \approx 13.56$ in.

3. (8 points each) Evaluate the following limits, if they exist.

(a) $\lim_{x\to 1} \dfrac{e^{x-1}-1}{x^2-1}$ indeterminate $0/0$

$$= \lim_{x\to 1} \frac{e^{x-1}}{2x}$$

$$= \frac{1}{2}$$

(b) $\lim_{x\to 0+} (\cos x)^{1/x}$ indeterminate 0^{∞}

Let $y = (\cos x)^{1/x}$

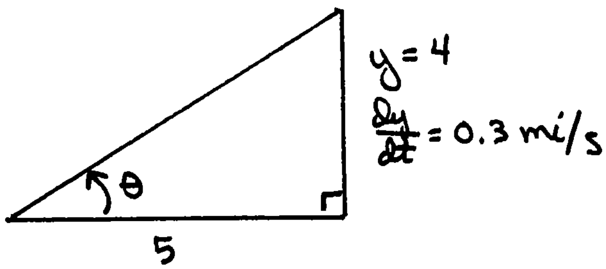
$$\ln y = \frac{1}{x}\ln(\cos x) = \frac{\ln(\cos x)}{x}$$

$\lim_{x\to 0^+} \dfrac{\ln(\cos x)}{x}$ indeterminate $0/0$

$$= \lim_{x\to 0^+} \frac{-\tan x}{1} = 0$$

Thus $\lim_{x\to 0^+} (\cos x)^{1/x} = e^0 = 1.$

4. (10 points) A camera tracks the launch of a vertically ascending spacecraft. The camera is located at ground level 5 miles from the launchpad. If the spacecraft is 4 miles up and traveling at 0.3 miles per second, at what rate is the camera angle (measured from the horizontal) changing?



$$\tan\theta = \frac{y}{5}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dy}{dt}$$

$$(1+\tan^2\theta)\frac{d\theta}{dt} = \frac{1}{5}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{\frac{1}{5}(0.3)}{1+(4/5)^2}$$

$$\approx 0.036585 \text{ radians/second}$$

5. (7 points each) Evaluate the following indefinite integrals.

(a) $\int \left(2x^{-1} + \frac{1}{\sqrt{x}}\right) dx, \qquad x \neq 0$

$$= 2\int \frac{1}{x}\,dx + \int x^{-1/2}\,dx$$

$$= 2\ln|x| + 2x^{1/2} + C$$

(b) $\int x^{2/3}(x^{-4/3} - 2)\,dx \quad = \int x^{-2/3} - 2x^{2/3}\,dx$

$$= \int x^{-2/3}\,dx - 2\int x^{2/3}\,dx$$

$$= 3x^{1/3} - \frac{6}{5}x^{5/3} + C$$

6. (10 points) Find the absolute extrema of the function $f(x) = x^2e^{-2x}$ on the interval $[0, 2]$. Classify each extremum as an absolute maximum or absolute minimum.

The absolute extrema will occur at the endpoints of the interval or at critical numbers inside the interval.

$$f'(x) = 2xe^{-2x} - 2x^2e^{-2x}$$

$$= 2xe^{-2x}(1-x)$$

$$0 = 2xe^{-2x}(1-x)$$

Critical numbers: $x = 0$, $x = 1$

$f(0) = 0 \leftarrow$ absolute minimum

$f(1) = e^{-2} \approx 0.135335 \leftarrow$ absolute maximum

$f(2) = 4e^{-4} \approx 0.073263$