Chapter 2 Review

#1. A function is continuous and differentiable, and has values given in the following table.

t	1	1.2	1.4	1.6	1.8
$h(t)$	.1	.3	.4	.8	.9

Fill in the table with approximate values of $h'(t)$

t	1	1.2	1.4	1.6	1.8
$h'(t)$					

Solution:
To approximate the derivative of a function you need to remember that the derivative function outputs the slope of the tangent line at a point on the graph. So, to approximate the derivative you approximate the slope of the tangent line. To approximate the slope of the tangent line find the slope of the secant line (sometimes these lines are called chords) connecting two points on the graph of the function. Doing this for the first two points we have that:

$$h'(1) \approx \frac{.3 - .1}{1.2 - 1} = \frac{.2}{.2} = 1.$$

Now, to approximate the derivative at 1.2 we can do it two different ways.

$$h'(1.2) \approx \frac{.3 - .1}{1.2 - 1} = \frac{.2}{.2} = 1 \text{ or } h'(1.2) \approx \frac{.4 - .3}{1.4 - 1.2} = .5.$$

Now, this is crucial, **the two above approximations to the derivative can be improved by averaging the two results**. Therefore, the best approximation to the derivative is given by:

$$h'(1.2) \approx .75$$

If we combine the process of finding the slope of the secant line and also taking the average we have that:

$$h'(1.4) \approx \frac{\frac{.8-.4}{.2} + \frac{.4-.3}{.2}}{2} = \frac{2 + .5}{2} = 1.25.$$

$$h'(1.6) \approx \frac{\frac{.9-.8}{.2} + \frac{.8-.4}{.2}}{2} = \frac{.5 + 2}{2} = 1.25.$$

For the last point in the table we can only use the slope of one secant line.

$$h'(1.8) \approx \frac{.9 - .8}{.2} = .5$$

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#2. Suppose $g(x)$ is an odd function and $g(2) = 3$ and $g'(3) = 4$. Find $g(-2)$ and $g'(-3)$.
Solution:

$$g(-2) = -3 \text{ and } g'(-3) = 4.$$

#3. The strength of a planet's magnetic field B, measured in Teslas, is function of its rotational speed ω, measured in units of radians per second. I.e, $B = f(\omega)$. Interpret the statement $f(.0017) = 1$, be sure to include units. Interpret the statement $f'(.0017) = 2$. Use the value of $f'(.0017)$ to estimate the value of $f(.002)$.

Solution:
$f(.0017) = 1$ means that if a planet is rotating at a speed of .0017 radians per second, the strength of its magnetic field is 1 Tesla. $f'(.0017) = 2$ means that if a planet is rotating at a speed of .0017 radians per second the strength of the magnetic field is increasing at a rate of 2 Teslas per radian per second. To figure out the units of the derivative function remember that $f'(\omega) = \dfrac{dB}{d\omega}$, this quantity has units of change in B divided by change in ω which is Teslas per radians per second. Now to estimate the value of $f(.002)$ we will approximate the function by the tangent line at .0017. We know the slope of this line is 2 and we know the point $(.0017, 1)$ lies on this line. Therefore, the equation of the line is:

$$y = 2(x - .0017) + 1.$$

Therefore, the approximate value of $f(.002)$ is given by:

$$f(.002) \approx y(.002) = 2(.002 - .0017) + 1 = .0006 + 1 = 1.0006.$$

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#5. Find the following limit. (Think about what this limit is!!).

$$\lim_{h \to 0} \frac{e^{2(3+h)} - e^{2(3)}}{h}$$

Solution:
Let $f(x) = e^{2x}$. Then,

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{e^{2(3+h)} - e^{2(3)}}{h}.$$

But, $f'(x) = 2e^{2x}$ which implies that $f'(3) = 2e^6$. Therefore we have that:

$$2e^6 = \lim_{h \to 0} \frac{e^{2(3+h)} - e^{2(3)}}{h}.$$

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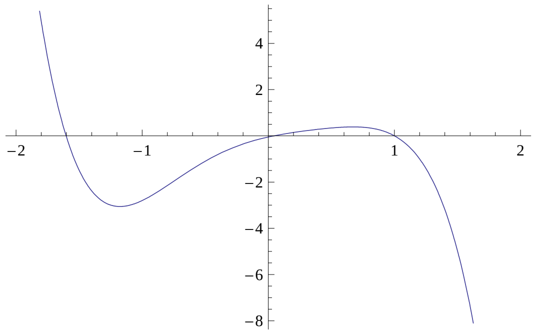
#6.. Sketch a graph of the derivative of the following function.



#7. Suppose the graph above is the derivative of some function $f(x)$. Estimate over what intervals $f(x)$ is increasing.

Solution: The function is increasing when the derivative is positive. So $f(x)$ is increasing on the intervals:

$$-\infty < x < -1.6 \text{ and } 0 < x < 1.$$