Note.

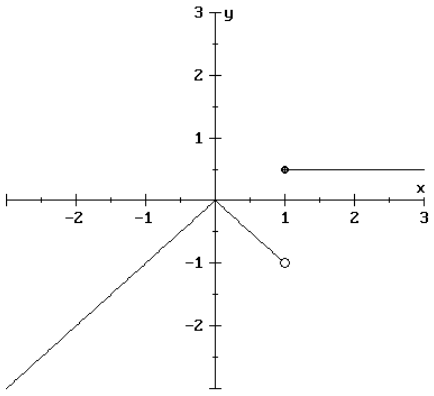
This is the third problem set of the Math 110/112 study aid. The formulas at the end of this problem set will be identical to the formulas given on the final exam.

Problem Set #3

1. If $f(x) = 2^x + 3x$, find $f(-2)$.

(A) $-\frac{23}{4}$ (B) -10 (C) $-\frac{5}{4}$ (D) -4 (E) None of these

2. Which of the following is the piecewise equation for the graph below?



(A) $f(x) = \begin{cases} -x^2 \text{ for } x < 1 \\ 1/2 \text{ for } x \geq 1 \end{cases}$ (B) $f(x) = \begin{cases} 1/2 \text{ for } x > 1 \\ |x| \text{ for } x < 1 \end{cases}$

(C) $f(x) = \begin{cases} -|x| \text{ for } x < 1 \\ 1/2 \text{ for } x \geq 1 \end{cases}$ (D) $f(x) = \begin{cases} 1/2 \text{ for } x > 1 \\ -|x| \text{ for } x < 1 \end{cases}$

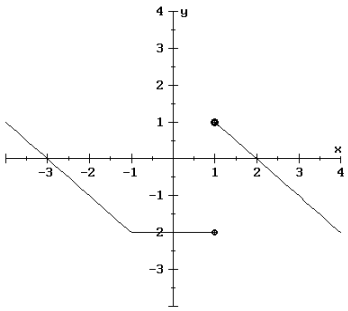
(E) None of these

3. Which of the following has a domain of all real numbers except 18?

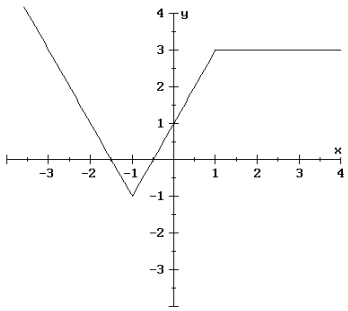
(1) $f(x) = \sqrt{x - 18}$ (2) $g(x) = \dfrac{2x}{x - 18}$ (3) $h(x) = \dfrac{1}{x^2 - 324}$

(A) 1 only (B) 2 only (C) 2 and 3 only (D) 1 and 2 only (E) All of them

4. Which of the following graphs represent y as a function of x?



(1)



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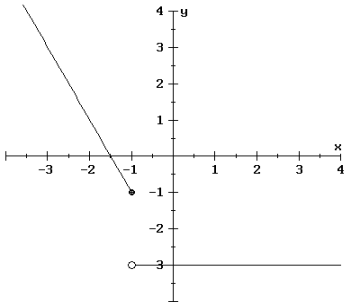
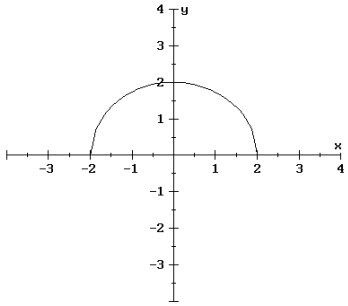


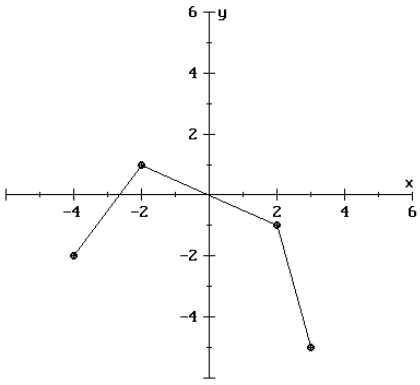
(3)



(4)

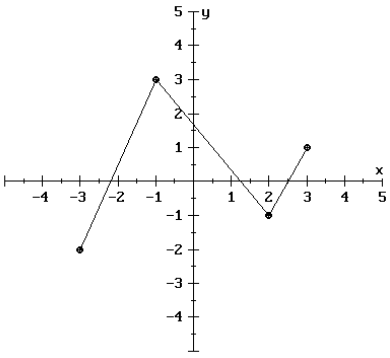
(A) All of them (B) None of them (C) 2, 3, and 4 only
(D) 2 and 3 only (E) 1 only

5. What is the RANGE of the function graphed below?



(A) $[-4, 3]$ (B) $[-5, -2]$ (C) $[-2, 3]$ (D) $[-5, 1]$ (E) None of these

6. Determine the intervals over which the function graphed below is increasing and decreasing.



Which of the following statements is TRUE about this function?

(A) The function is **increasing** on $(-2, 3)$; **decreasing** on $(-1, 3)$

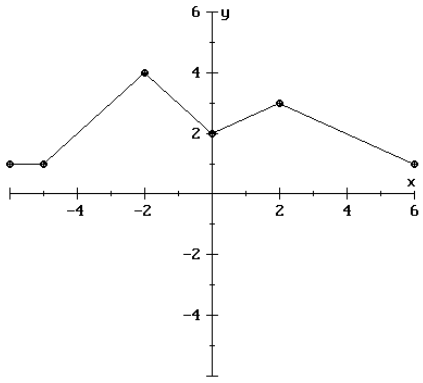
(B) The function is **increasing** on $(-3, -1)$ and $(2, 3)$; **decreasing** on $(-1, 2)$

(C) The function is **increasing** on $(-1, 2)$; **decreasing** on $(-3, -1)$ and $(2, 3)$

(D) The function is **increasing** on $(-1, 1)$; **decreasing** on $(-1, 2)$

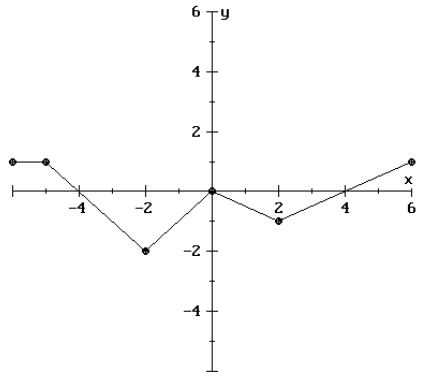
(E) The function is **increasing** on $(-3, 0)$ and $(1, 3)$; **decreasing** on $(0, 1)$

7. The graph of $y = f(x)$ is shown below



This is the graph of $y = f(x)$

The graph below is a transformation of the graph of $y = f(x)$. Which of the following is the formula for the function graphed below?



(A) $y = f(-x) - 2$ (B) $y = -f(x) - 2$ (C) $y = -f(x + 2)$

(D) $y = -f(x) + 2$ (E) $y = f(-x + 2)$

8. If $(5, -6)$ is a point on the graph of $y = g(x)$, which of the following must be a point on the graph of $y = -g(x) + 1$?

(A) $(-5, -6)$ (B) $(-5, 7)$ (C) $(5, 6)$ (D) $(5, 7)$ (E) $(-6, 6)$

9. Given $f(x) = 4x + 1$ and $g(x) = |5x + 2|$, find $\left(\frac{f}{g}\right)(-4)$.

(A) $-\frac{15}{22}$ (B) $\frac{17}{22}$ (C) $-\frac{5}{6}$ (D) -4 (E) None of these

10. Given $f(x) = 3 + 2x$ and $h(x) = \sqrt{x}$, find $(f \circ h)(x)$.

(A) $\sqrt{3 + 2x}$ (B) $3 + 2\sqrt{x}$ (C) $\sqrt{3} + \sqrt{2x}$

(D) $3\sqrt{x} + 2x\sqrt{x}$ (E) None of these

11. Search and Rescue teams are used in remote areas in the West to find lost people. Experience has shown the team's chance of finding an individual is a function of the distance by which team members are separated. The percentage found for various separation distances is shown in the table below.

Separation distance (ft)	Percent found
20	90
40	80
60	70
80	60
100	50

Find an equation to express the percent found, P, as a function of the separation distance, d, of the team members.

(A) $P = 0.5d + 100$ (B) $P = 290 - 0.5d$ (C) $P = 100 - 0.5d$

(D) $P = 90 - 5d$ (E) $P = 2d + 50$

12. Which of the following lines is PARALLEL to $3x - 4y = 7$?

(A) $y = 3x - 7$

(B) $y = -\frac{3}{4}x + 8$

(C) $y = -\frac{4}{3}x - 2$

(D) $y = -\frac{4}{3}x - 3$

(E) None of these

13. For the function $f(x) = -x^2 + 6x + 62$, which of the following statements is/are true?

(1) the x-coordinate of the vertex is -3

(2) $f(x)$ has a maximum value of 71

(3) one of the x-intercepts is $\left(3 + \sqrt{71}, 0\right)$

(A) 1 and 2 only (B) 1 and 3 only (C) 2 and 3 only

(D) 2 only (E) 1 only

14. Find the vertex of the quadratic function

$$f(x) = 3x^2 + 7x + 4.$$

The vertex is located:

(A) Above the x-axis (B) Below the x-axis

(C) At the origin (D) On the x-axis but not at the origin

(E) None of these

15. Which of the following has two x-intercepts?

(1) $f(x) = x^2 - 40$ (2) $g(x) = -x^2 + 60x - 903$

(3) $h(x) = -(x + 125)^2 + 1$

(A) 1 only (B) 2 and 3 only (C) 2 only

(D) 1 and 3 only (E) None of these

16. For the years 1975 through 1990, the average price, p (in dollars per million British thermal units), of fuel used to generate electricity in the U.S. can be modeled by the function

$$p(t) = -0.021t^2 + 0.50t - 1.04$$

where t is time in years since 1970. Estimate the maximum average price, p, of fuel used according to this model.

(A) \$11.9 per million *Btu* (B) \$1.94 per million *Btu* (C) \$2.1 per billion *Btu*

(D) \$10.2 per million *Btu* (E) None of these

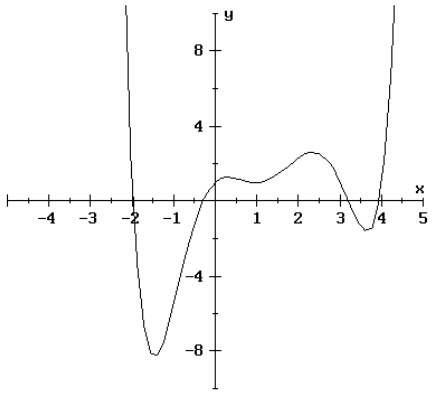
17. For the function $f(x) = -3(x - a)^5(x + b)(x - c)^2$, where a, b and c are positive real numbers, which of the following is/are correct?

(1) $f(x)$ is a polynomial of degree 5 (2) $f(x)$ has a zero at $x = b$

(3) $f(x)$ has an x-intercept at $(c, 0)$ (4) $y \to -\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to \infty$

(A) 1 and 2 only (B) 2 and 4 only (C) 1 and 3 only (D) 3 and 4 only

18. Consider the polynomial function below. Which of the following statement(s) is/are true about this polynomial?



(1) This polynomial could have degree 6

(2) This polynomial could have degree 4

(3) This polynomial could have degree 8

(A) 1 and 2 only (B) 1 and 3 only (C) 1 only

(D) 2 only (E) 2 and 3 only

19. For which of the following functions does $y \to \infty$ as $x \to -\infty$?

(1) $y = 4x^3 - 3x$

(2) $y = 2x^4 - x + 10$

(3) $y = -x^6 + 3x^5 - 2$

(A) None of them (B) 2 only (C) 2 and 3 only

(D) 1 and 3 only (E) 3 only

20. Find the value of 'b' so that $x + 2$ is a factor of $P(x) = x^4 - (b + 1)x^2 - 5bx - 9b$.

(A) $b = -2$ (B) $b = 4$ (C) $b = 2$ (D) $b = -4$ (E) None of these

21. Factor $T(x) = x^3 + x^2 + 13x + 30$. One of the factors is:

(A) $x + 3$ (B) $x - 5$ (C) $x + 2$ (D) $x - 10$ (E) None of these

22. If 3 is a zero of $f(x) = 18x^3 - 111x^2 + 161x + 30$, what are the other real zeros of $f(x)$? The SMALLEST real zero is:

(A) $-\frac{16}{5}$ (B) $\frac{16}{5}$ (C) $-\frac{1}{6}$ (D) $-\frac{1}{5}$ (E) 0

23. Find a polynomial of lowest degree that has $-3, -\sqrt{2}$, and $\sqrt{2}$ as its zeros.

(A) $x^3 + 3x^2 - 2x - 6$ (B) $x^2 + (-3 - \sqrt{2})x - 6$

(C) $x^3 + 3x^2 - 4x - 12$ (D) $x^2 - 2\sqrt{2x} + 3$

(E) None of these

24. Find all the asymptotes of the rational function $y = \frac{2x^2 + 1}{2x + 3}$

(A) $x = -\frac{3}{2}, \quad y = x - \frac{3}{2}$ (B) $x = -\frac{3}{2}, \quad y = 1$ (C) $x = \frac{3}{2}, \quad y = x + 3$

(D) $x = \frac{3}{2}, \quad y = 1$ (E) $x = -\frac{3}{2}, \quad y = x + \frac{1}{2}$

25. Which of the following rational functions has no vertical asymptote?

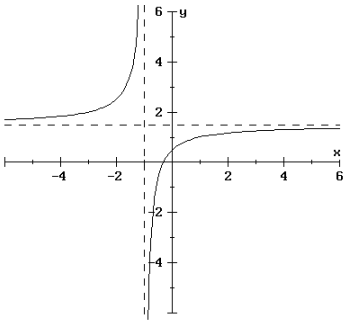
(1) $y = \frac{2}{x^2 - 5}$ (2) $y = \frac{5x - 4}{x^2 + 1}$ (3) $y = \frac{7x^2}{-x^2 + 3}$

(A) 1 and 3 only (B) 1 and 2 only (C) 2 and 3 only

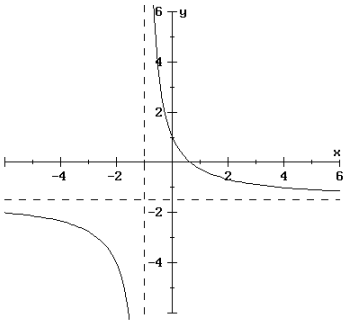
(D) 2 only (E) 1 only

26. Which of the following is the graph of $y = \dfrac{2x+1}{2x-3}$

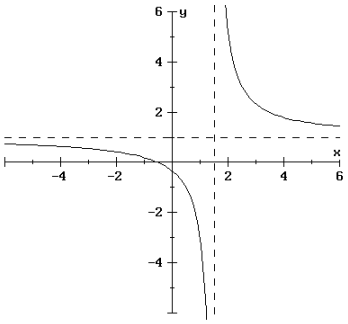
(A)



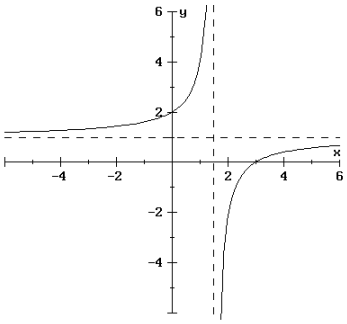
(B)



(C)



(D)



(E) None of these

27. Which ONE of the following rational functions has a graph that:

(i) Has vertical asymptotes at $x = 20$ and $x = -20$

(ii) Has an x-intercept $(1, 0)$

(iii) Has a horizontal asymptote at $y = 0$

(A) $f(x) = \dfrac{x^2 - 400}{x + 1}$ (B) $f(x) = \dfrac{x + 1}{x^2 - 20}$ (C) $f(x) = \dfrac{x - 1}{x^2 - 400}$

(D) $f(x) = \dfrac{x + 1}{x^2 - 20}$ (E) None of these

28. Find the domain and range of the function $H(x) = 3^{x+2} - 4$.

(A) Domain: $(-2, \infty)$ Range: $(-4, \infty)$

(B) Domain: $(-\infty, \infty)$ Range: $(-4, \infty)$

(C) Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

(D) Domain: $(2, \infty)$ Range: $(4, \infty)$

(E) Domain: $(-2, \infty)$ Range: $(4, \infty)$

29. Consider the functions $f(x)$ and $g(x)$ represented by the tables shown below.

x	$f(x)$
2	4
3	3
4	6
5	5

x	$g(x)$
2	4
3	5
4	3
5	4

Which, if either, of these functions is one-to-one?

(A) both $f(x)$and $g(x)$ (B) $g(x)$only (C) $f(x)$only

(D) neither $f(x)$nor $g(x)$

30. If $h(x) = 20x - 62$, what is $h(h^{-1}(6))$?

(A) 2 (B) 58 (C) −6 (D) 6 (E) None of these

31. Find $R^{-1}(x)$ if $R(x) = \dfrac{C}{3x-1}$ (C is a real number)

(A) $R^{-1}(x) = \dfrac{C}{3}x + C$ (B) $R^{-1}(x) = \dfrac{3x-1}{C}$

(C) $R^{-1}(x) = \dfrac{C+x}{3x}, x \neq 0$ (D) $R^{-1}(x) = \dfrac{C-3x}{x}, x \neq 0$

(E) None of these

32. Suppose $g(x)$ is the inverse of the function $f(x)$. Which of the following tables would be correct for $g(2x)$ if $f(x)$ is given in the table below.

x	–2	–1	0	1	2
$f(x)$	1	3	5	7	9

(A)

x	1	3	5	7	9
$g(2x)$	–4	–2	0	2	4

(B)

x	1	3	5	7	9
$g(2x)$	–1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1

(C)

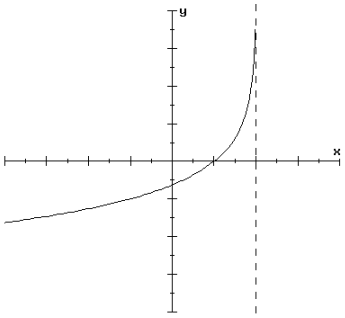
x	2	6	10	14	18
$g(2x)$	–2	–1	0	1	2

(D)

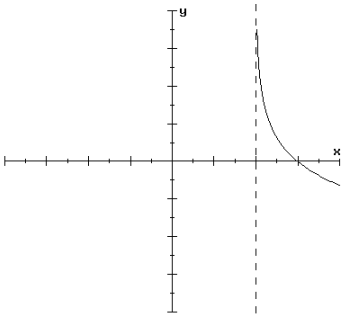
x	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
$g(2x)$	–2	–1	0	1	2

33. Which of the following most resembles the graph of $y = -\log_3(x - a)$, where a is a positive constant?

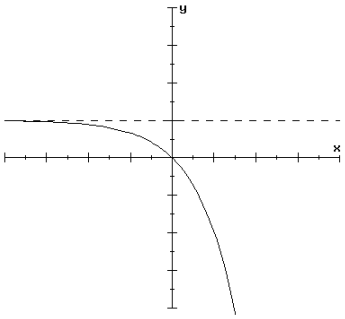
(A)



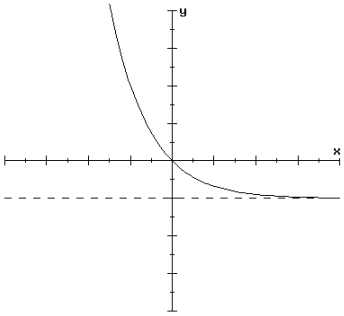
(B)



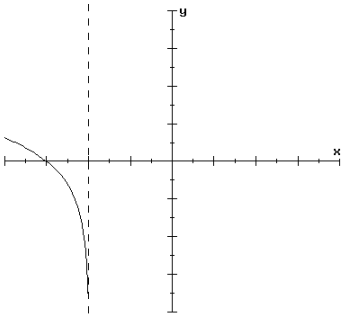
(C)



(D)



(E)



34. For the function $R(x) = \log_2 x$, which of the following is/are correct?

(1) The domain is $[0, \infty)$ (2) The range is $(-\infty, \infty)$ (3) $R(x)$ is one-to-one

(A) 2 and 3 only (B) 1 and 2 only (C) 3 only

(D) 1 and 3 only (E) All of them

35. Express as a single logarithm and simplify if possible:

$$\frac{1}{3}\log_a x + 4\log_a y - 2\log_a z$$

(A) $\log_a[x^{1/3} + y^4 - z^2]$ (B) $\log_a\left[\frac{1}{3}x + 4y - 2z\right]$ (C) $\frac{7}{3}\log_a\left[\frac{xy}{z}\right]$

(D) $\log_a\left[\frac{x^{1/3}y^4}{z^2}\right]$ (E) None of these

36. If M and N are positive, which of the following is/are correct?

(1) $\ln(MN) = \ln M + \ln N$ (2) $\ln(M + N) = \ln M + \ln N$

(3) $\frac{\ln M}{\ln N} = \ln M - \ln N$

(A) All of them (B) None of them (C) 1 only

(D) 1 and 3 only (E) 2 and 3 only

37. Use natural logarithms to solve for x : $3 + 6e^{2x} = 5$

(A) $x = \frac{1}{2}\ln 3$ (B) $x = 2\ln 3$ (C) $x = \frac{1}{2}\ln\left(\frac{1}{3}\right)$

(D) $x = \ln 3 - 2$ (E) None of these

38. Solve for x : $\log_3 x - \log_3(x-1) = 2$

(A) $\frac{1}{8}$ (B) $\frac{9}{8}$ (C) $\frac{10}{9}$ (D) $\frac{100}{99}$ (E) None of these

39. In 1980, the population of the United States was approximately 226.5 million people. In 1990, the population had grown to approximately 246.7 million. Assuming an exponential growth model $A = Pe^{rt}$, what is the projected population of the U.S. in the year 2000?

(A) Less than 260 million

(B) Between 260 million and 265 million

(B) Between 265 million and 270 million

(B) Between 270 million and 275 million

(B) More than 275 million

40. How much MORE money will you earn in an account that compounds interest continually than in an account that compounds interest quarterly if you invest $3000 for 7 years at an interest rate of 11%?

(A) $67.02 (B) $59.37 (C) $101.16 (D) $32.52 (E) None of these

41. A computer virus has been introduced into a system of computers. The data below was collected over the next 5 months:

Number of computers infected	Months since virus was introduced
24	1
96	2
384	3
1536	4
6144	5

Which of the following equations best describes the number of computers infected, y, as a function of the time, t (measured in months)?

(A) $y = 6(4^t)$ (B) $y = 24(4^t)$ (C) $y = 24 + 72t$

(D) $y = 24 + t$ (E) None of these

42. Based on the pattern of the data from the previous question, find the *y*-intercept of this function, and describe its practical significance.

(A) (0,6); there were no computers infected at time $t = 6$

(B) (0,6); 6 computers were initally infected

(C) (0,4); there were no computers infected at time $t = 4$

(D) (0,4); 4 computers were initially infected

43. Given that $\sum_{k=1}^{12}(2k^2 + ck) = 1456$, determine the value of c.

(A) 4 (B) 0.5 (C) 1.5 (D) 3 (E) 2

44. Simplify: $\dfrac{(n+1)!}{2 \cdot n!}$

(A) $\dfrac{n+1}{2}$ (B) $\dfrac{1}{2n(n-1)}$ (C) $\dfrac{(n+1)(n-1)}{2}$

(D) $\dfrac{1}{n}$ (E) None of these

45. Find the 5*th* term of a sequence if $a_n = \begin{cases} 3 & \text{if } n = 1 \\ 2a_{n-1} + 1 & \text{if } n = 2,3,4,\ldots \end{cases}$

(A) 63 (B) 11 (C) 42 (D) 54 (E) None of these

46. For an arithmetic sequence, if $d = 3$ and $a_{39} = 125$, find a_1.

(A) 9 (B) 12 (C) 11 (D) 8 (E) None of these

47. How many bricks will there be in a wall one brick in thickness if there are 39 bricks in the bottom row, 37 bricks in the second row, 35 bricks in the third row, and so forth, to the top row, which has one brick?

(A) 300 (B) 420 (C) 250 (D) 270 (E) None of these

48. Determine if the following sequence is arithmetic or geometric:

$$3, 7, 11, 15, \bullet\bullet\bullet$$

Find the 51st term of the sequence.

(A) 609 (B) 195 (C) 203 (D) 243 (E) None of these

49. Determine if the following sequence is arithmetic or geometric or neither:

$$\frac{1}{3}, \ -\frac{2}{3}, \ \frac{4}{3}, \ -\frac{8}{3}, \ldots$$

Find the sum of the first 6 terms of the sequence.

(A) $\frac{11}{3}$ (B) -7 (C) $\frac{20}{3}$ (D) -28 (E) $-\frac{19}{3}$

50. Determine the sum of the infinite geometric series, if it exists:

$$72 - 36 + 18 - 9 + \ldots$$

(A) 36 (B) 48 (C) 72 (D) 144 (E) Does not exist

FORMULAS

$A = P\left(1 + \frac{r}{n}\right)^{n\,t}$ $\qquad$ $A = Pe^{r\,t}$

$a_n = a_1 + (n-1)d$ $\qquad$ $a_n = a_1 r^{n-1}$

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$ $\qquad$ $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right),\ r \neq 1$ $\qquad$ $\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1 - r},\ |r| < 1$

Summation Formulas:

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\qquad$ $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$