Math 110/112 Final Exam Study Aid

Note.

This study aid is intended to help you review for the final exam. It covers the primary concepts in the course. It is separated into 3 problem sets, each of which contains 50 questions that review concepts covered throughout the semester. Although the final exam will be similar to the study aid, it will not be identical to it. You should also review tests, notes, study aids and homework given during the semester. The formulas at the end of the third problem set will be identical to the formulas given on the final exam.

Problem Set #1

1. Which of the following equations determine y as a function of x?

(1) $3x + 2y^3 = 10$ (2) $\sqrt{x-1} + y = 8$ (3) $2x - y^2 - 7 = 0$

(4) $3x^2 - xy = 1$

(A) All of them (B) 1 and 3 only (C) 1, 2 and 4 only

(D) 1 and 2 only (E) 1 and 4 only

2. If $g(x) = \begin{cases} 1 - 3x & \text{for } x < -1 \\ 3 - x^2 & \text{for } x \geq -1 \end{cases}$, what is $g(-3)$?

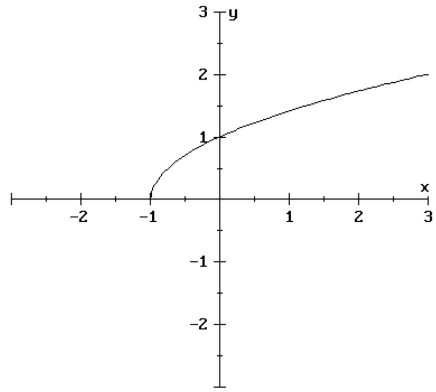
(A) 12 (B) 10 (C) –8 (D) –6 (E) None of these

3. What is the DOMAIN of the function $f(x) = 12 - \sqrt{108 - 3x}$?

(A) $(-\infty, 36]$ (B) $[-108, 108]$ (C) $[36, \infty)$

(D) $(-\infty, 36) \cup (36, \infty)$ (E) None of these

4. What is the DOMAIN of the function represented by the graph below?



(A) $(-\infty, \infty)$ (B) $[0, \infty)$ (C) $[-1, \infty)$ (D) $(-\infty, -1]$ (E) None of these

5. Express the area of a rectangle AS A FUNCTION OF ITS WIDTH if the width is 25% of its length. Let L and W represent length and width, respectively.

(A) $A = (0.25W)(W)$ (B) $A = (0.75W)(W)$ (C) $A = (4W)(W)$

(D) $A = 4LW$ (E) None of these

For the following TWO questions, use the partial table of values for the function $y = f(x)$ shown below:

x	$f(x)$
-3	9
-2	
-1	-6
1	
2	0
3	

6. Complete the table above so that $f(x)$ is an ODD function.
The missing values, in order, are:

(A) $0, 6, -9$ (B) $0, -6, 9$ (C) $0, -\frac{1}{6}, \frac{1}{9}$

(D) $0, \frac{1}{6}, -\frac{1}{9}$ (E) Cannot be determined

7. Complete the table above so that $f(x)$ is an EVEN function.
The missing values, in order, are:

(A) $0, 6, -9$ (B) $0, -6, 9$ (C) $0, -\frac{1}{6}, \frac{1}{9}$

(D) $0, \frac{1}{6}, -\frac{1}{9}$ (E) Cannot be determined

8. The relation that vertically compresses the graph of $y = \sqrt{x}$ and shifts the graph up twenty units is:

(A) $y = \frac{5}{3}\sqrt{x} + 20$ (B) $y = \sqrt{\frac{7}{2}x + 20}$ (C) $y = \frac{3}{4}\sqrt{x} + 20$

(D) $y = 2\sqrt{x + 20}$ (E) None of these

9. You can get the graph of $y = -f(2x)$ by transforming the graph of $y = f(x)$ in the following way:

(A) Compress horizontally and reflect across the x-axis

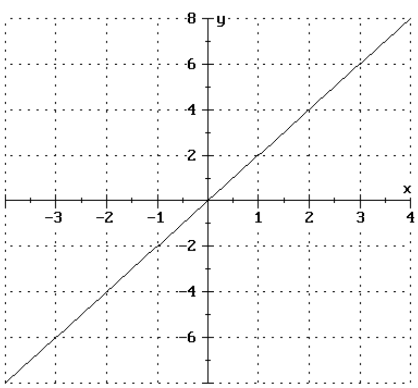
(B) Compress horizontally and reflect across the y-axis

(C) Expand vertically and reflect across the x-axis

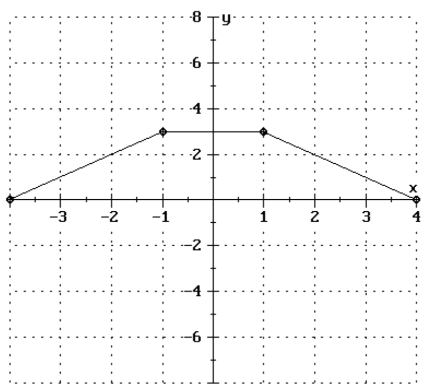
(D) Expand vertically and reflect across the y-axis

(E) None of these

Use the graphs below to answer the next TWO questions.



This is the graph of $y = f(x)$



This is the graph of $y = g(x)$

10. Using the previous graphs, find $(f - g)(3)$.

(A) 3 (B) 5 (C) 15 (D) 6 (E) Not enough information

11. Using the previous graphs, find $(f \circ g)(1)$.

(A) 6 (B) 2 (C) 1 (D) 3 (E) None of these

12. Find the equation of the line passing through the points $(2, 1)$ and $(4, 7)$.

The slope and y-intercept are:

	slope:	y-intercept
(A)	3	(0,7)
(B)	$\frac{1}{3}$	$(0, \frac{1}{3})$
(C)	$\frac{1}{3}$	$(0, \frac{5}{3})$
(D)	3	(0,–5)

(E) None of these

13. What is the degree and leading coefficient of

$$f(x) = -3x^2 + 6x^4 - 4x^5 + 7?$$

(A) degree is 2, leading coefficient is –3

(B) degree is 11, leading coefficient is –4

(C) degree is 5, leading coefficient is –4

(D) degree is 5, leading coefficient is 4

(E) None of these

14. Write $f(x) = 3x^2 + 60x - 1$ in standard form. The SUM of the x and y coordinates of the vertex is:

(A) –473 (B) 365 (C) –365 (D) 473 (E) None of these

15. Find the vertex of the quadratic function $f(t) = \frac{4}{7}t^2 - \frac{16}{7}t + 3$.

The y-coordinate of the vertex is:

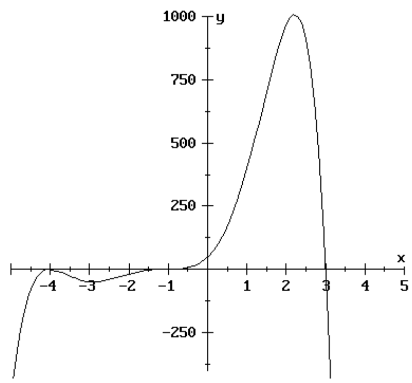
(A) $\frac{1}{2}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) 1 (E) None of these

16. A horticulturist has determined that the number of inches a young oak tree grows in one year is a function of the annual rainfall, r, given by $g(r) = -0.01r^2 + 0.1r + 2$. What is the maximum number of inches a young oak can grow in a year? The maximum number of inches is:

(A) less than 1 (B) between 1 and 2 (C) between 2 and 3

(D) between 3 and 4 (E) between 4 and 5

17. Which of the following could be the equation of the polynomial $P(x)$ graphed below?



(A) $P(x) = (x-4)^2(x+3)^2(x-1)$ (B) $P(x) = (x+4)^3(x-3)(x+1)$

(C) $P(x) = -(x+4)^2(x-3)(x+1)^3$ (D) $P(x) = (x-4)^2(x+3)(x+1)$

(E) $P(x) = -(x-4)^2(x+3)(x-1)^3$

18. Find all the real zeros of $f(x) = x^3 + 5x^2 + 7x + 2$. The LARGEST real zero is:

(A) $\dfrac{-3+\sqrt{5}}{2}$ (B) -0.5 (C) $\dfrac{-3+\sqrt{13}}{2}$

(D) $\dfrac{-3+\sqrt{7}}{2}$ (E) -2

19. Which of the following statements is/are equivalent to:
"$x+3$ is a factor of the polynomial $f(x)$"?

(1) $x = 3$ is a solution of $f(x) = 0$

(2) $x = -3$ is a zero of $f(x)$

(3) $(-3, 0)$ is an x-intercept of $f(x)$

(A) 1 only (B) 2 only (C) 3 only

(D) 1 and 3 only (E) 2 and 3 only

20. What is the remainder when $5x^3 - 6x^2 + 3$ is divided by $x^2 - x + 4$?

(A) $-21x + 7$ (B) 7 (C) $21x - 7$

(D) $-14x$ (E) None of these

21. The number of miles per gallon, M, for an experimental engine is given by

$$M = \frac{2000x}{1000 + x^2} + 5$$

where x is the speed of the car in miles per hour, $10 \le x \le 60$. Using your calculator, determine the speed that yields the greatest number of miles per gallon.

(A) about 60 mph (B) about 10 mph (C) about 37 mph

(D) about 32 mph (E) There is no maximum

22. Are the zeros of $p(x) = x^4 + (3/4)x^3 - (1/2)x^2 + (1/4)x + 5$ the same as the zeros of $q(x) = 4x^4 + 3x^3 - 2x^2 + x + 20$?

(A) No (B) Yes

23. Which of the following MUST be true?

(1) A polynomial of degree 4 has four unique zeros.

(2) A polynomial of degree 5 has at least 1 real zero.

(3) A polynomial of degree 2 has at least 1 rational zero.

(A) 1 only (B) 2 only (C) 3 only

(D) 1 and 2 only (E) 1 and 3 only

24. Find the vertical asymptote(s), if any, for $f(x) = \dfrac{x+1}{4x^2-1}$.

(A) $x = \frac{1}{4}$ (B) $x = \frac{1}{2}$ (C) $x = -\frac{1}{2}, x = \frac{1}{2}$

(D) $x = -\frac{1}{4}, x = \frac{1}{4}$ (E) $x = -1$

25. Find the value of the parameter a so that the rational function $g(x) = \dfrac{ax^2+3x+7}{x+1}$ has the slant asymptote $y = 2x+1$.

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

26. Which of the following statement(s) is/are true about the function $f(x) = \dfrac{6x+1+0.05x^2}{x-2}$

(1) $y \to 6$ as $x \to \infty$

(2) $f(x)$ has exactly one x-intercept

(3) $f(x)$ has a slant asymptote

(A) 1 and 2 only (B) 2 and 3 only (C) 2 only (D) 3 only (E) 1 only

27. Which ONE of the following statements is true about the rational function $g(x) = \dfrac{3x^2 - x}{3x^2 + x}$?

(A) The graph of g has a zero at $x = 0$.

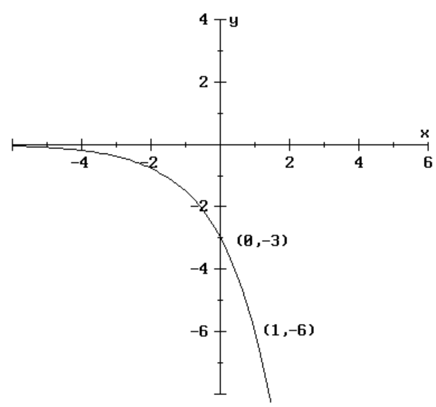
(B) The graph of g has a vertical asymptote at $x = 0$.

(C) The graph of g has a hole at $x = 0$.

(D) The graph of g has a horizontal asymptote at $y = 0$.

(E) The graph of g has no horizontal asymptote.

28. The graph below represents $y = C(a)^x$. Find the values of C and a.



(A) $C = -3, a = 2$ (B) $C = -3, a = 3$ (C) $C = 2, a = -2$

(D) $C = 2, a = -3$ (E) $C = \frac{1}{3}, a = -2$

29. If $f(x)$ is a one-to-one function, and $f(2) = 7$, then which of the following CANNOT be true?

(A) $f(7) = 2$ (B) $f^{-1}(7) = 2$ (C) $f^{-1}(5) = 3$

(D) $f(-2) = 4$ (E) $f(-2) = 7$

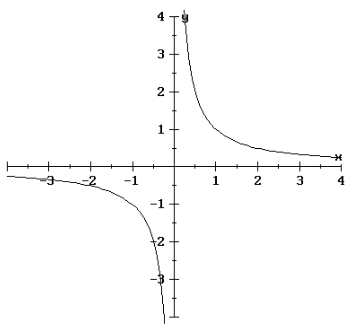
30. Which of the following functions is/are one-to-one?

(1) $f(x) = \dfrac{6}{x - 13}$ (2) $g(x) = 0.05(x + 3)$ (3) $h(x) = \sqrt{3x^2 - 40}$

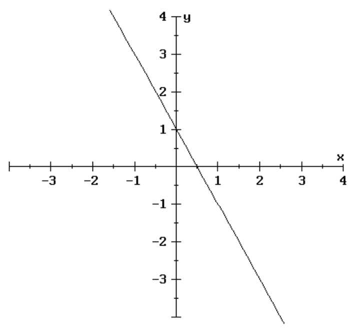
(A) 2 and 3 only (B) 1 and 2 only (C) 1, 2 and 3 only

(D) 1 and 3 only (E) None are one-to-one

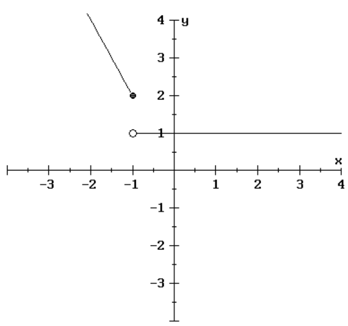
31. Of the functions graphed below, which have inverse functions?



(1)



(2)



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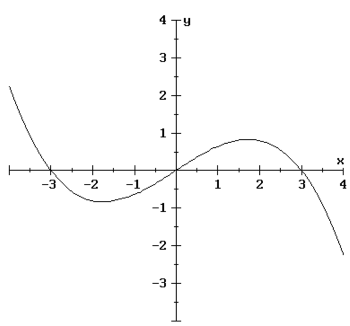


(4)

(A) 1, 2 and 3 only (B) 2 only (C) 1 and 2 only

(D) 1, 2 and 4 only (E) None of the answers A-D

32. Which of the following is/are correct? $[a > 0, a \neq 1]$

(1) $\log_a 1 = 0$ (2) $\log_a 0 = 0$ (3) $\ln 1 = e$ (4) $\ln e^a = a$

(A) None of them

(B) 4 only

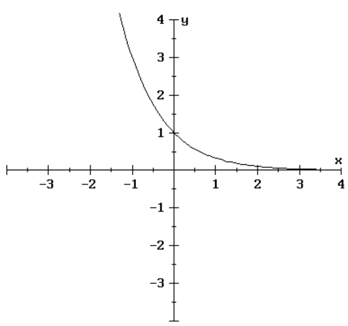
(C) 1 and 4 only

(D) 3 only

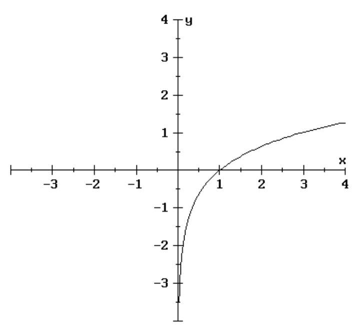
(E) 2 and 4 only

33. Which of the following is a graph of the INVERSE of $y = \log_3 x$?

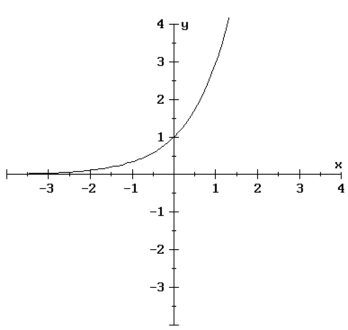
(A)



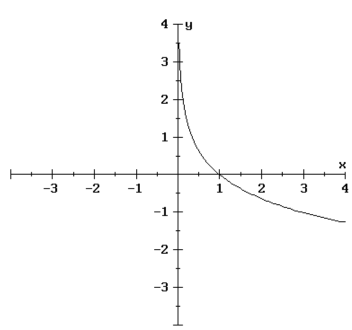
(B)



(C)



(D)



(E) None of these

34. Find the x-intercept of the graph of $M(x) = \log_6(2x+3)$.

(A) $(1.5, 0)$ (B) $(0, 0)$ (C) $(-3, 0)$
(D) $(0.5, 0)$ (E) $(-1, 0)$

35. Rewrite $23^b = a$ in logarithmic form.

(A) $\log_a b = 23$ (B) $\log_{23} a = b$ (C) $\log_{23} b = a$
(D) $\log_b 23 = a$ (E) None of these

36. Find the exact value of $\ln\left[\sqrt[4]{e^5}\right]$.

(A) 0.8 (B) 1.25 (C) e (D) $0.8e$ (E) None of these

37. Solve $\log_5 x = 2$ and $\log_2 32 = w$. The two solutions are:

(A) $x = \sqrt{5}$ and $w = \sqrt{32}$ (B) $x = 25$ and $w = 5$ (C) $x = 32$ and $w = 16$
(D) $x = 25$ and $w = 16$ (E) None of these

38. Solve for x : $3^x = 5^{x-1}$
The solution is a number:

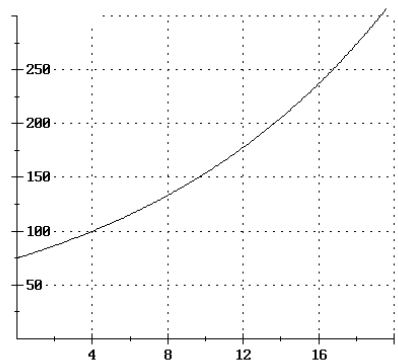
(A) between 2 and 4 (B) between -5 and -3 (C) between -1 and 0
(D) between -3 and -1 (E) None of these

39. Solve for x : $\ln(2x - 1) = 2$

(A) $x = \frac{e^2}{2}$ (B) $x = \frac{e^2 + 1}{2}$ (C) $x = e^2 + \frac{1}{2}$ (D) $x = \frac{e^4}{2}$

(E) None of these

It has been rumored that college costs have been growing exponentially. Suppose the cost of four years of college (in thousands of dollars) can be expressed as $y = Ce^{kt}$ where $t = 0$ corresponds to 1992, $t = 4$ corresponds to 1996, and so on. The graph of this function is shown below. Use this graph to answer the next TWO questions.



40. Use the graph (above) to find the value of C.

(A) 1992 (B) 75 (C) 0 (D) 100 (E) None of these

41. What is the value of k?

(A) 0.288 (B) 0.333 (C) 0.072 (D) 0.066 (E) None of these

42. The release of fluorocarbons used in household sprays destroys the ozone layer in the upper atmosphere. Suppose the amount of ozone is given by $P = Ce^{-0.0025t}$ where t is measured in years. How long will it take for 70% of the ozone to disappear? (Round to the nearest yr.)

(A) About 143 yrs. (B) About 1699 yrs. (C) About 1360 yrs.

(D) About 482 yrs. (E) None of these

43. If $\sum_{k=1}^{25} a_k = 40$ and $\sum_{k=1}^{25} b_k = 125$, find $\sum_{k=1}^{25}(3a_k - b_k + 2)$.

(A) –35 (B) 45 (C) –83 (D) 3 (E) None of these

44. Using the formula $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, find $\sum_{k=1}^{20}(2k^2 + 1)$.

(A) 2870 (B) 2890 (C) 4250 (D) 5780 (E) 5760

45. If the *nth* term of a sequence is given by $a_n = \frac{(-1)^{n-1}2^n}{(n+1)!}$,what is the fifth term?

(A) $-\frac{1}{24}$ (B) $\frac{2}{45}$ (C) $\frac{5}{3}$ (D) $-\frac{17}{121}$ (E) None of these

46. Write the terms of the sum: $\sum_{k=2}^{8} \frac{1}{k^2}$.

The third term is:

(A) $\frac{1}{9}$ (B) $\frac{1}{16}$ (C) $\frac{1}{25}$ (D) $\frac{1}{36}$ (E) None of these

47. For an arithmetic sequence, if $a_1 = \frac{3}{4}$ and $a_8 = -\frac{11}{4}$, what is d?

(A) $-\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $-\frac{3}{8}$ (D) $-\frac{3}{2}$ (E) None of these

48. An arithmetic series has $a_1 = 2$ and $a_5 = 18$. Find the sum of the first 20 terms.

(A) 87 (B) 780 (C) 800 (D) 1680 (E) None of these

49. Find the sum: $\sum_{n=1}^{9}(-2)^n$

(A) –1028 (B) –1266 (C) –342 (D) –482 (E) None of these

50. Determine if the following sequence is arithmetic or geometric or neither:

$$2, 7, 12, 17, \ldots$$

Find the sum of the first 20 terms of the sequence.

(A) 2020 (B) 97 (C) 1980 (D) 99 (E) None of these