1 Solutions to Monday's Review Session

1.1 What is the domain of the functions $f(x) = \frac{1}{x-3}$ and $g(x) = \frac{\sqrt{x+2}}{x-3}$.

A lot of times when you are doing a domain problem you want to ask yourself where the function is undefined. For $f(x)$, we could have a problem of dividing by zero. So, we must have that $x \neq 3$. Therefore, the domain for $f(x)$ is simply:

$$D: \; x \neq 3$$

Now, for $g(x)$ we have the added problem that we cannot take the square root of a negative number. Therefore we must make sure that everything underneath the radical is positive or greater than zero. Written in math symbols this means that:

$$\begin{aligned} x+2 &\geq 0 \\ \Rightarrow x &\geq -2. \end{aligned}$$

Also, we have that $x \neq 3$ since we still don't want to divide by zero. Therefore, the domain for $g(x)$ is given by

$$D: \; x \geq -2 \text{ and } x \neq 3.$$

1.2 If $f(x) = \sqrt{x}$, evaluate and completely simplify the difference quotient: $\frac{f(x+h)-f(x)}{h}$.

The goal with any difference quotient problem is to eliminate the h in the denominator. What makes this problem difficult is the square root. To "get rid" of the square root I am going to multiply the top and bottom of the fraction by the conjugate.

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{x+h}-\sqrt{x}}{h} \\ &= \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \\ &= \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} \\ &= \frac{h}{h(\sqrt{x+h}+\sqrt{x})} \\ &= \frac{1}{\sqrt{x+h}+\sqrt{x}}. \end{aligned}$$

1.3 Find a possible formula for the following tables:

x	0	1	2	3	4
f(x)	7	12	17	22	27

x	0	1	2	3	4
g(x)	3	5.1	8.67	14.739	25.89151

The first function is linear. To check if a table is a linear function check if it has a constant slope. The first table has a constant slope of 5. Also, the table has a y-intercept of 3. Therefore, the formula for the function is given by:

$$f(x) = 5x + 3.$$

The second table is exponential. The way you check if a table is exponential is to take the quotient (i.e. divide) of consecutive y values and see if you get a constant. For this table we have that

$$\begin{aligned} 5.1/3 &= 1.7 \\ 8.67/5.1 &= 1.7 \\ 14.739/8.67 &= 1.7 \\ 25.89151/14.739 &= 1.7. \end{aligned}$$

Therefore, this table is an exponential function.

Now, lets see why this works. Suppose we have a function in the form $g(x) = ab^x$ that fits the data points of some table. Now, lets see what happens if we divide two consecutive y-values. For $x = 2$ and $x = 1$ we have that

$$g(2)/g(1) = (ab^2)/(ab^1) = b$$

Now, lets pick an arbitrary input n. The next consecutive input is $n + 1$. If we then divide there corresponding outputs we have that

$$g(n+1)/g(n) = (ab^{n+1})/(ab^n) = b.$$

It follows that no matter what two consecutive points you pick their quotient will always be the constant b.

So, if we return to our table above we have that $b = 1.7$ and $a = 3$, which is the y-intercept. So, the equation for $g(x)$ is simply

$$g(x) = 3(1.7)^x.$$

Also, recall that $b = 1 + r$ where r is the decimal representation of the percent rate of change. Therefore, every time x increases by 1, $g(x)$ increases by 70%.

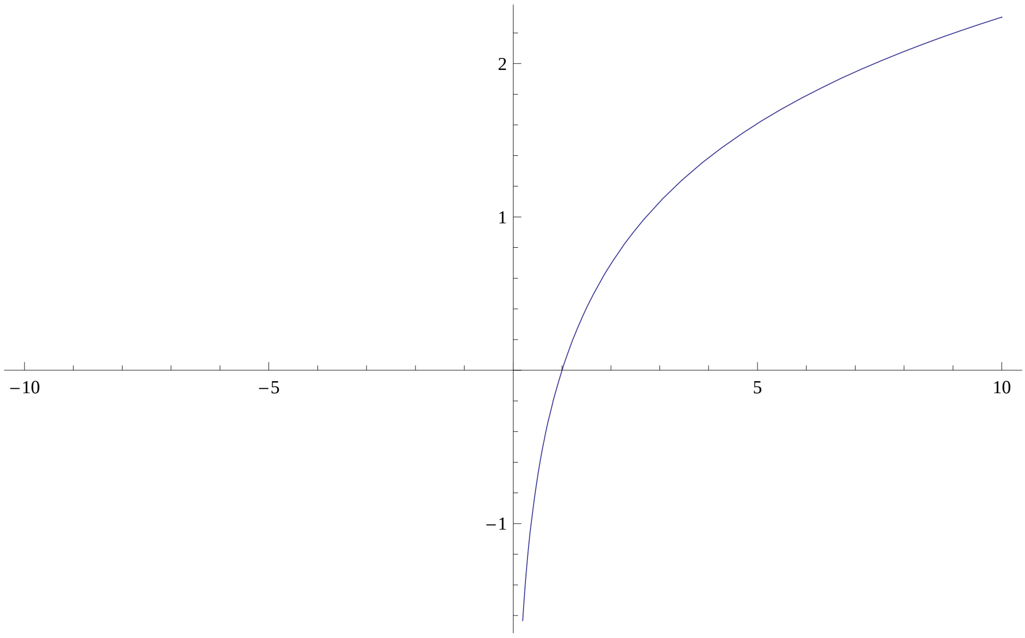
Finally, lets convert our exponential function into the form $g(x) = ae^{kx}$. We already know that $a = 3$, so all we need to do is find k. Well, since this is just another way to represent the same exponential function we must have that for all values of x the following must be true:

$$\begin{aligned} 3(1.7)^x &= 3e^{kx} \\ \Rightarrow \ln(1.7^x) &= \ln(e^{kx}) \\ \Rightarrow x\ln(1.7) &= kx \\ \Rightarrow \ln(1.7) &= k. \end{aligned}$$

Remember that k is called the continuous growth rate.

1.4 What is the range and domain of the function $f(x) = 3 + 2\ln(x-4)$.

First, **you cannot use your calculator to do this problem**. One way to do this problem is to think about this is as a transformation of the base graph $g(x) = \ln(x)$. The graph of $g(x) = ln(x)$ is given below: Now, $ln(x)$ has a range of $(-\infty, \infty)$ and a domain of $(0, \infty)$. Now, $f(x)$ just is this



graph shifted right 4, vertically expanded by a factor of 2, and then vertically shifted up 3. This means that the range gets shifted to $(-\infty, \infty)$ (no change) and the domain is shifted to $(4, \infty)$.

Another way to do this problem is to think about the inverse of $f(x)$. The inverse of $f(x)$ is given by

$$f^{-1}(x) = e^{1/2(x-3)} + 4.$$

Now, the domain of $f^{-1}(x)$ is easily seen to be all real numbers since you can input any number. Now, the range of $f^{-1}(x)$ is easy to find since this is basically an exponential function that is shift up vertically 4 units. Therefore, the range is $(4, \infty)$. This means that $f(x)$ must have the opposite domain and range. In this case this means that the domain of $f(x)$ is $(4, \infty)$ and the range of $f(x)$ is $(-\infty, \infty)$.

Finally, the **only** asymptote of $f(x)$ is the line $x = 4$.

1.5 If $(2,3)$ lies on the graph of $f(x)$. What point must lie on the graph of $g(x) = -2 \cdot f(x-4) + 3$.

This is a simple transformation problem. The graph of $g(x)$ is related to the graph of $f(x)$ by first shifting right 4, vertically expanding by 2, reflecting across the x-axis, and then shifting up 3. Therefore, $(2,3)$ gets translated to the point $(6,-3)$.

Another way to think about this problem is to use our knowledge that $f(2) = 3$. So if we evaluate $g(6)$ we have that

$$g(6) = -2 \cdot f(2) + 3 = -2 \cdot 3 + 3 = -3.$$

This means that the point $(6,-3)$ lies on the graph of $g(x)$.

1.6 Determine which of the following functions are odd, even, or neither.

- $f(x) = \frac{x^2+1}{x^6-1}$

  $$f(-x) = \frac{(-x)^2+1}{(-x)^6-1} = \frac{x^2+1}{x^6-1} = f(x).$$

  This function is even.

- $f(x) = \frac{|x|}{x-2}$.

  $$f(-1) = 1/(-3) = -1/3.$$
  $$f(1) = 1/(-1) = -1.$$

  This function is neither even or odd.

- $f(x) = \sin(x)\cos(x)$.

  $$f(-x) = \sin(-x)\cos(-x) = -\sin(x)\cos(x) = -f(x).$$

  This function is odd.