Math-110 College Algebra
Fall 2007

Note: Show all work. Correct answers without support will receive at most half credit. Incorrect answers without support will receive no credit.

Name: _____

Quiz 9 Solutions and Take Home Quiz

#1. The population of Japanese beetles in central Pennsylvania triples every month in the summer. In June, the population of Japanese beetles in a small town in central Pennsylvania was 200 million.

1. Determine the population of beetles in July and August. (There is no need to use a formula for this problem.)
2. Determine a formula for A.
3. After how many months will the insect population be 800 million (Round to two decimal places).

Solutions:

1. Since the population triples every month we know that there will be 600 million beetles in July and 1800 beetles in August.
2. Since the population of beetles is growing exponentially we want to use the formula $A(t) = Pe^{rt}$, where t is the number of months after June, $P = 200$million is the initial population of the beetles, and r is the growth factor. To determine the value of r we use the fact that after 1 month there will be 600 million beetles. This means that $A(1) = 600$. Therefore,

$$\begin{aligned} & 600 = 200e^{r\cdot 1} \\ \Rightarrow\ & 3 = e^{r} \\ \Rightarrow\ & \ln(3) = r \\ \Rightarrow\ & r = 1.0986 \end{aligned}$$

   Therefore $A(t) = 200e^{1.0986t}$.

3. To determine when the population will be 800 million we need to solve the equation:

$$\begin{aligned} & 800 = 200e^{1.0986t} \\ \Rightarrow\ & 4 = e^{1.0986t} \\ \Rightarrow\ & \ln(4) = 1.0986t \\ \Rightarrow\ & t = \frac{\ln(4)}{1.0986} \\ \Rightarrow\ & t = 1.26. \end{aligned}$$

   So, after 1.26 months the there will be 800 beetles.

#2. A type of tinted glass reduces the intensity of the light by 20% per 3 millimeters of thickness. Write a function I that models the intensity of sunlight passing through a pane of this glass that is x millimeters thick, given that I_0 is the intensity of the sunlight entering the glass.

Solution:

Since, the light is being reduced by 20% for every 3 millimeters of glass we again want to use an exponential model of the form $I = I_0 e^{rx}$, where x is the thickness of the glass, r is the rate of decay, and I_0 is the initial intensity of the sunlight entering the glass. What we know about this problem is that if the thickness of the glass is 3 millimeters than the intensity of the light leaving the glass is 80% of the amount that entered. In mathematical symbols, this is the quantity $.80I_0$. Therefore, using the formula $I = I_0 e^{rx}$, we have that:

$$\begin{aligned} & I(3) = .80I_0 \\ \Rightarrow \quad & .80I_0 = I_0 e^{r\cdot 3} \\ \Rightarrow \quad & .80 = e^{3r} \\ \Rightarrow \quad & \ln(.8) = 3r \\ \Rightarrow \quad & \frac{\ln(.8)}{3} = r \\ \Rightarrow \quad & r = -.074. \end{aligned}$$

Therefore, the function we are looking for is given by

$$I = I_0 e^{-.074x}.$$

Take Home Quiz Questions

#1. In 1980 the population of Pennsylvania was 11,863,895 and in 1990 the population was 11,881,643. Assuming an exponential growth model of the form $A = Pe^{rt}$, what is the projected population of the state in 2010?

#2. Suppose the amount of drinkable water W left in Tucson is given by $W(t) = Ie^{-.003t}$, where t is measured in years. How long will it take for only 30% of the original amount of water to be left?