1 Solutions to the Second Algebra Worksheet

1.1 Solve $e^{2x} - 2e^x - 3 = 0$ for x.

This is a quadratic problem. Let $z = e^x$. Then, we have that:

$$
\begin{aligned}
z^2 - 2z - 3 &= 0 \\
\Rightarrow (z-3)(z+1) &= 0 \\
\Rightarrow z = 3 \text{ or } z &= -1 \\
\Rightarrow e^x = 3 \text{ or } e^x &= -1
\end{aligned}
$$

$$\Rightarrow x = \ln(3) \text{ or } x = \ln(-1)$$

$$\Rightarrow x = \ln(3).$$

1.2 Solve $\frac{1}{2}x - 8x^{1/3} = 0$ for x.

This is a factoring problem.

$$
\begin{aligned}
\frac{1}{2}x - 8x^{1/3} &= 0 \\
\Rightarrow x^{1/3}\left(\frac{1}{2}x^{2/3} - 8\right) &= 0
\end{aligned}
$$

$$\Rightarrow x^{1/3} = 0 \text{ or } \frac{1}{2}x^{2/3} - 8 = 0$$

$$\Rightarrow x = 0 \text{ or } x^{2/3} = 16$$

$$\Rightarrow x = 0 \text{ or } x = 16^{3/2}$$

$$\Rightarrow x = 0 \text{ or } x = 4^3$$

$$\Rightarrow x = 0 \text{ or } x = 64.$$

1.3 Solve $2^x = 3e^{4x}$ for x.

With problems involving things being raised to the x power it is a good idea to try to use logs. Just be sure to use the properties correctly!!

$$
\begin{aligned}
2^x &= 3e^{4x} \\
\Rightarrow \ln(2^x) &= \ln(3e^{4x}) \\
\Rightarrow x\ln(2) &= \ln(3) + \ln(e^{4x}) \\
\Rightarrow x\ln(2) &= \ln(3) + 4x \\
\Rightarrow x\ln(2) - 4x &= \ln(3) \\
\Rightarrow x(\ln(2) - 4) &= \ln(3) \\
\Rightarrow x &= \frac{\ln(3)}{\ln(2) - 4}.
\end{aligned}
$$

1.4 Solve $1 - \frac{1}{1+x} = k$ for x.

Whenever what you are solving for appears in the denominator of a fraction it is a good idea to "clear" the fraction by multiplying by the denominator.

$$\begin{aligned}
1 - \frac{1}{1+x} &= k \\
\Rightarrow (1+x)\left(1 - \frac{1}{1+x}\right) &= k(1+x) \\
\Rightarrow 1 + x - 1 &= k + kx \\
\Rightarrow x &= k + kx \\
\Rightarrow x - kx &= k \\
\Rightarrow x(1-k) &= k \\
\Rightarrow x &= \frac{k}{1-k}
\end{aligned}$$

1.5 Solve $e^{x-2} = k4^x$ for x.

$$\begin{aligned}
e^{x-2} &= k4^x \\
\ln(e^{x-2}) &= \ln(k4^x) \\
x - 2 &= \ln(k) + \ln(4^x) \\
x - 2 &= \ln(k) + x\ln(4) \\
x - x\ln(4) &= \ln(k) + 2 \\
x(1 - \ln(4) &= \ln(k) + 2 \\
x &= \frac{\ln(k) + 2}{1 - \ln(4)}.
\end{aligned}$$

1.6 Solve $4x^3e^{kx} - 16xe^{kx} = 0$ for x.

$$\begin{aligned}
4x^3e^{kx} - 16xe^{kx} &= 0 \\
4xe^{kx}(x^2 - 4) &= 0 \\
4xe^{kx}(x-2)(x+2) &= 0
\end{aligned}$$

$$4x = 0 \text{ or } \text{ or } e^{kx} = 0 \text{ or } x - 2 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = 0 \text{ or } \text{ or } kx = \ln(0) \text{ or } x = 2 \text{ or } x = -2$$

$$\Rightarrow x = 0 \text{ or } x = 2 \text{ or } x = -2.$$

1.7 Solve $2\ln(5x) = \ln(x+2)$ for x.

Here we want to undo the logarithms. But, first we will have to use a properties of logarithms to get everything inside of a logarithm.

$$\begin{aligned}
2\ln(5x) &= \ln(x+2) \\
\Rightarrow \ln\left((5x)^2\right) &= \ln(x+2) \\
\Rightarrow e^{\ln(25x^2)} &= e^{\ln(x+2)} \\
\Rightarrow 25x^2 &= x+2 \\
\Rightarrow 25x^2 - x - 2 &= 0 \\
\Rightarrow x &= \frac{1 \pm \sqrt{1+4(25)(2)}}{50} \\
\Rightarrow x &= \frac{1 \pm \sqrt{201}}{50}
\end{aligned}$$

Now, a tricky thing about this problem is that there is only one solution. Remember, any solution we find must satisfy the original equation. If we try to substitute in the solution taking the negative sign we will have a negative logarithm. Therefore, the only solution is given by:

$$x = \frac{1+\sqrt{201}}{50}.$$

1.8 Solve $(2x+5)^2 + 5(2x+5) - 36 = 0$ for x.

This is another quadratic problem. I am going to directly factor it instead of going through the process of substituting like I did in problem 1.

$$\begin{aligned}
(2x+5)^2 + 5(2x+5) - 36 &= 0 \\
((2x+5)+9)\,((2x+5)-4) &= 0
\end{aligned}$$

$$2x + 14 = 0 \text{ or } 2x + 1 = 0$$

$$\Rightarrow 2x = -14 \text{ or } 2x = -1$$

$$\Rightarrow x = -7 \text{ or } x = -\frac{1}{2}.$$

1.9 Solve $10^{2x} + 3(10^x) - 10 = 0$, for x

.

$$\begin{aligned}
10^{2x} + 3(10^x) - 10 &= 0 \\
(10^x + 5)(10^x - 2) &= 0
\end{aligned}$$

$$10^x + 5 = 0 \text{ or } 10^x - 2 = 0$$

$$\Rightarrow 10^x = -5 \text{ or } 10^x = 2$$

$$\Rightarrow x = \log(-5) \text{ or } x = \log(2)$$

$$\Rightarrow x = \log(2).$$

1.10 Solve $3(2^{2t}) - 11(2^t) - 4 = 0$, for x.

$$\begin{aligned} 3(2^{2t}) - 11(2^t) - 4 &= 0 \\ \Rightarrow (3(2^t) + 1)(2^t - 4) &= 0 \end{aligned}$$

$$\Rightarrow 3(2^t) + 1 = 0 \text{ or } 2^t - 4 = 0$$

$$\Rightarrow 3(2^t) = -1 \text{ or } 2^t = 4$$

$$\Rightarrow 2^t = \frac{-1}{3} \text{ or } 2^t = 2^2$$

$$\Rightarrow \ln(2^t) = \ln\left(\frac{-1}{3}\right) \text{ or } t = 2$$

$$\Rightarrow t = 2$$

1.11 Solve $\sqrt{x+9} - 2 = \sqrt{x-3}$ for x.

These type of problems can be a real pain. The goal is to get the x out from under the square root. The obvious way to do that is to first square both sides. But, this will not get rid of all of the square roots. The second step is to then isolate the remaining square root term and then square both sides again.

$$\begin{aligned} \sqrt{x+9} - 2 &= \sqrt{x-3} \\ \Rightarrow (\sqrt{x+9} - 2)^2 &= x - 3 \\ \Rightarrow x + 9 - 4\sqrt{x+9} + 4 &= x - 3 \\ \Rightarrow -4\sqrt{x+9} &= -16 \\ \Rightarrow \sqrt{x+9} &= 4 \\ \Rightarrow x + 9 &= 16 \\ \Rightarrow x &= 7. \end{aligned}$$

1.12 Solve $\frac{x}{12} - \frac{2}{x} = \frac{1}{x}$ for x.

$$\begin{aligned} \frac{x}{12} - \frac{2}{x} &= \frac{1}{x} \\ \Rightarrow 12x\left(\frac{x}{12} - \frac{2}{x}\right) &= \frac{1}{x}12x \\ \Rightarrow x^2 - 24 &= 12 \\ \Rightarrow x^2 &= 36 \\ \Rightarrow x &= \pm 6. \end{aligned}$$

1.13 Solve $\left(\frac{1}{x+8}\right)^2 + \frac{1}{x+8} - 6 = 0$ for x.

$$\begin{aligned}
\left(\frac{1}{x+8}\right)^2 + \frac{1}{x+8} - 6 &= 0 \\
\Rightarrow \left(\frac{1}{x+8} + 3\right)\left(\frac{1}{x+8} - 2\right) &= 0
\end{aligned}$$

$$\frac{1}{x+8} + 3 = 0 \text{ or } \frac{1}{x+8} - 2 = 0$$

$$\Rightarrow (x+8)\left(\frac{1}{x+8} + 3\right) = 0(x+8) \text{ or } (x+8)\left(\frac{1}{x+8} - 2\right) = 0(x+8)$$

$$\Rightarrow 1 + 3(x+8) = 0 \text{ or } 1 - 2(x+8) = 0$$

$$\Rightarrow 1 + 3x + 24 = 0 \text{ or } 1 - 2x - 16 = 0$$

$$\Rightarrow x = \frac{-25}{3} \text{ or } x = \frac{-15}{2}.$$

1.14 Solve $x^4 + 2x^2 = 3$ for x.

$$\begin{aligned}
x^4 + 2x^2 &= 3 \\
x^4 + 2x^2 - 3 &= 0 \\
(x^2+3)(x^2-1) &= 0
\end{aligned}$$

$$x^2 + 3 = 0 \text{ or } x^2 - 1 = 0$$

$$\Rightarrow x^2 = -3 \text{ or } x^2 = 1$$

$$\Rightarrow x = \pm\sqrt{-3} \text{ or } x = \pm 1$$

$$\Rightarrow x = \pm 1.$$

1.15 Solve $\frac{1}{x-1} + 4 = \frac{1}{5}$ for x.

$$\begin{aligned}
\frac{1}{x-1} + 4 &= \frac{1}{5} \\
\Rightarrow 5(x-1)\left(\frac{1}{x-1} + 4\right) &= 5(x-1)\frac{1}{5} \\
\Rightarrow 5 + 20(x-1) &= x - 1 \\
\Rightarrow 5 + 20x - 20 &= x - 1 \\
\Rightarrow 19x = 14 & \\
\Rightarrow x = \frac{14}{19}. &
\end{aligned}$$

1.16 Solve $(x+5)(x-2) = 8$ for x.

$$\begin{aligned} (x+5)(x-2) &= 8 \\ \Rightarrow (x+5)(x-2) - 8 &= 0 \\ \Rightarrow x^2 + 5x - 2x - 10 - 8 &= 0 \\ \Rightarrow x^2 + 3x - 18 &= 0 \\ \Rightarrow (x+6)(x-3) &= 0 \end{aligned}$$

$$x = -6 \text{ or } x = 3.$$

1.17 Solve $3az + 1 = 3a - 4z$

$$\begin{aligned} 3az + 1 &= 3a - 4z \\ \Rightarrow 3az + 4z &= 3a - 1 \\ \Rightarrow z(3a+4) &= 3a - 1 \\ \Rightarrow z &= \frac{3a-1}{3a+4}. \end{aligned}$$

1.18 Solve $\frac{12}{z} - \frac{7}{z+1} = 1$ for z.

$$\begin{aligned} \frac{12}{z} - \frac{7}{z+1} &= 1 \\ z(z+1)\left(\frac{12}{z} - \frac{7}{z+1}\right) &= z(z+1) \\ 12(z+1) - 7z &= z(z+1) \\ 12z + 12 - 7z &= z^2 + z \\ 0 &= z^2 - 4z - 12 \\ 0 &= (z-6)(z+2) \end{aligned}$$

$$z = 6 \text{ or } z = -2.$$

1.19 Solve $0 = 12y^2 + 12y - 24$ for y.

$$\begin{aligned} 0 &= 12y^2 + 12y - 24 \\ \Rightarrow 0 &= 12(y^2 + y - 2) \\ \Rightarrow 0 &= y^2 + y - 2 \\ \Rightarrow 0 &= (y+2)(y-1) \end{aligned}$$

$$y = -2 \text{ or } y = 1.$$

1.20 Solve $\ln(w+2) = \ln(w) + \ln(5)$ for w.

$$\begin{aligned}
\ln(w+2) &= \ln(w) + \ln(5) \\
\Rightarrow \ln(w+2) &= \ln(5w) \\
\Rightarrow e^{\ln(w+2)} &= e^{\ln(5w)} \\
\Rightarrow w+2 &= 5w \\
\Rightarrow w &= 1/2.
\end{aligned}$$

1.21 Solve $P = 10e^{kt}$ for t.

$$\begin{aligned}
P &= 10e^{kt} \\
\Rightarrow \frac{P}{10} &= e^{kt} \\
\Rightarrow \ln\left(\frac{P}{10}\right) &= \ln(e^{kt}) \\
\Rightarrow \ln\left(\frac{P}{10}\right) &= kt \\
\Rightarrow \frac{\ln\left(\frac{P}{10}\right)}{k} &= t.
\end{aligned}$$

1.22 Solve $2^t = e^{t-2}$ for t.

$$\begin{aligned}
2^t &= e^{t-2} \\
\Rightarrow \ln(2^t) &= \ln(e^{t-2}) \\
\Rightarrow t\ln(2) &= t-2 \\
\Rightarrow t\ln(2) - t &= -2 \\
\Rightarrow t\left(\ln(2)-1\right) &= -2 \\
\Rightarrow t &= \frac{-2}{\ln(2)-1}.
\end{aligned}$$

1.23 Solve $t^2e^{3t} + 9te^{3t} = 0$ for t.

$$\begin{aligned}
t^2e^{3t} + 9te^{3t} &= 0 \\
\Rightarrow te^{3t}(t+9) &= 0
\end{aligned}$$

$$t = 0 \text{ or } e^{3t} = 0 \text{ or } t+9 = 0$$

$$\Rightarrow t = 0 \text{ or } t = -9.$$

1.24 Solve $\ln(t) + 2t = \ln(8)$ for t.

This is not solvable.

1.25 Solve $5\log(t) = 3$ for t.

$$\begin{aligned} 5\log(t) &= 3 \\ \Rightarrow \log(t) &= \frac{3}{5} \\ \Rightarrow 10^{\log(t)} &= 10^{\frac{3}{5}} \\ \Rightarrow t &= 10^{\frac{3}{5}}. \end{aligned}$$

1.26 Solve $t^4 - 9t = 0$ for t

$$\begin{aligned} t^4 - 9t &= 0 \\ \Rightarrow t(t^3 - 9) &= 0 \end{aligned}$$

$$t = 0 \text{ or } t^3 - 9 = 0$$
$$\Rightarrow t = 0 \text{ or } t = 9^{1/3}.$$

1.27 Solve $\frac{(n^2-2)(3n+5)}{n-3} = 0$ for n.

$$\begin{aligned} \frac{(n^2-2)(3n+5)}{n-3} &= 0 \\ \Rightarrow (n-3)\frac{(n^2-2)(3n+5)}{n-3} &= 0(n-3) \\ \Rightarrow (n^2-2)(3n+5) &= 0 \end{aligned}$$

$$n^2 - 2 = 0 \text{ or } 3n + 5 = 0$$
$$n = \pm\sqrt{2} \text{ or } n = \frac{-5}{3}$$

1.28 Solve $\frac{(p^2-5)(p+3)^2}{p+1} = 0$ for p.

$$\begin{aligned}
\frac{(p^2-5)(p+3)^2}{p+1} &= 0 \\
\Rightarrow (p+1)\frac{(p^2-5)(p+3)^2}{p+1} &= 0(p+1) \\
\Rightarrow (p^2-5)(p+3)^2 &= 0
\end{aligned}$$

$$p^2 - 5 = 0 \text{ or } (p+3)^2 = 0$$
$$\Rightarrow p^2 = 5 \text{ or } p+3 = 0$$
$$\Rightarrow p = \pm\sqrt{5} \text{ or } p = -3.$$

1.29 Solve $\log(w) + \log(w+1) = \log(20)$ for w

$$\begin{aligned}
\log(w) + \log(w+1) &= \log(20) \\
\Rightarrow \log(w(w+1)) &= \log(20) \\
\Rightarrow 10^{\log(w(w+1))} &= 10^{\log(20)} \\
\Rightarrow w(w+1) &= 20 \\
\Rightarrow w^2 + w - 20 &= 0 \\
\Rightarrow (w+5)(w-4) &= 0
\end{aligned}$$

$$w = -5 \text{ or } w = 4$$

But, you cannot take the log of a negative number so when we substitute these value back into the original equation we have that the only solution is:

$$w = 4.$$

1.30 Solve $T = \frac{1}{2\pi}\sqrt{\frac{L}{g}}$ for L.

$$\begin{aligned}
T &= \frac{1}{2\pi}\sqrt{\frac{L}{g}} \\
\Rightarrow T^2 &= \frac{1}{4\pi^2}\frac{L}{g} \\
\Rightarrow 4\pi^2 g T^2 &= L.
\end{aligned}$$

1.31 Solve $\frac{(1-2\mu\theta)(\mu+\theta)-(\theta-\mu\theta^2)}{(\mu+\theta^2)^2} = 0$ for θ.

This problem looks like it is hard and it is hard. But, it is important that you go through this one.

$$
\begin{aligned}
\frac{(1-2\mu\theta)(\mu+\theta)-(\theta-\mu\theta^2)}{(\mu+\theta^2)^2} &= 0 \\
\Rightarrow (\mu+\theta^2)^2\left(\frac{(1-2\mu\theta)(\mu+\theta)-(\theta-\mu\theta^2)}{(\mu+\theta^2)^2}\right) &= 0(\mu+\theta^2)^2 \\
\Rightarrow (1-2\mu\theta)(\mu+\theta)-(\theta-\mu\theta^2) &= 0 \\
\Rightarrow \mu-2\mu^2\theta+\theta-2\mu\theta^2-\theta+\mu\theta^2 &= 0 \\
\Rightarrow \mu-\mu\theta^2-2\mu^2\theta &= 0 \\
\Rightarrow \mu(1-\theta^2-2\mu\theta) &= 0 \\
\Rightarrow 1-\theta^2-2\mu\theta &= 0 \\
\Rightarrow \theta^2+2\mu\theta-1 &= 0 \\
\Rightarrow \theta = \frac{-2\mu \pm \sqrt{4\mu^2+4}}{2} & \\
\Rightarrow \theta = \frac{-2\mu \pm 2\sqrt{\mu^2+1}}{2} & \\
\Rightarrow \theta = \frac{2(-\mu \pm \sqrt{\mu^2+1}}{2} & \\
\Rightarrow \theta = -\mu \pm \sqrt{\mu^2+1}. &
\end{aligned}
$$

1.32 This problem was done in class.

1.33 Don't worry about this problem

1.34 Don't worry about this problem

1.35 Don't worry about this problem

1.36 Don't worry about this problem

1.37 Simplify $\frac{2x^{2/3}(x^2-3)2x-\frac{2}{3}x^{-1/3}(x^2-3)^2}{(x^2-3)^{3/2}}$.

$$\begin{aligned}
\frac{2x^{2/3}(x^2-3)2x-\frac{2}{3}x^{-1/3}(x^2-3)^2}{(x^2-3)^{3/2}} &= \frac{2x^{-1/3}(x^2-3)(x^1\cdot 2x-\frac{1}{3}(x^2-3))}{(x^2-3)^{3/2}} \\
&= \frac{2x^{-1/3}(2x^2-\frac{1}{3}x^2+1)}{(x^2-3)^{1/2}} \\
&= \frac{2(\frac{5}{3}x^2+1)}{x^{1/3}(x^2-3)^{1/2}}.
\end{aligned}$$

1.38 Simplify $\frac{1-\frac{1}{x}}{\frac{1}{x}-x}$.

$$\begin{aligned}
\frac{1-\frac{1}{x}}{\frac{1}{x}-x} &= \frac{\frac{x-1}{x}}{\frac{1-x^2}{x}} \\
&= \frac{x-1}{x}\cdot\frac{x}{1-x^2} \\
&= \frac{x-1}{1-x^2} \\
&= \frac{x-1}{(1-x)(1+x)} \\
&= \frac{x-1}{-(x-1)(1+x)} \\
&= -\frac{1}{1+x}.
\end{aligned}$$