APMA: 0200
Homework #6

Due Date: November 13, 2015

Be sure that all phase portraits are large and detailed. Label all x-intercepts, y-interpcepts and null-clines

In this homework the goal is to develop and analyze your own models of the spread of infectious diseases. This is a very open ended assignment. In many cases there is not a "correct" answer. You will be assessed on the feasibility of your models and your analysis.

1. In the following bullet points you will be tasked with adapting the SIR model to model different effects. In all cases analyze your model by drawing appropriate phase portraits and analyzing the long term behavior of the disease.

   - Adapt the SIR model to account for the possibility that some infected individuals may be become susceptible again.
   - Adapt the SIR model to account for a policy of quarantining in which the rate of quarantining increases with the number of infected individuals. Is it possible to prevent an epidemic?
   - Adapt the SIR model to account for a policy of both quarantining and vaccinating against a disease. Is it possible to prevent an epidemic.

2. The so-called SIS model of an infectious disease is the following:

$$\frac{dS}{dt} = -bSI + rI,$$
$$\frac{dI}{dt} = bSI - rI,$$

   where as usual S denotes the susceptible population, I the infected and $b, r > 0$.

   - Develop and analyze a modified version of this model that incorporates population growth by assuming that only susceptible individuals give birth to other susceptible individuals. What does this model tell us a about the long term behavior of the disease? **Hint:** It is OK to consider population growth without a carrying capacity.
   - Develop and analyze another model that incorporates population growth but now assume that individuals who are infected give birth to children who are susceptible. What does this model tell us a about the long term behavior of the disease?
   - Develop and analyze another model that incorporates population growth but now assume that individuals who are infected give birth to children who are also infected. What does this model tell us a about the long term behavior of the disease?

3. Some people who have had an infectious disease such as tuberculosis never completely recover and continue to carry the infection, whilst not suffering the disease themselves. In this case the individuals become carriers for the disease and can still infect others. The most famous example of this is probably Mary Mallon (1869-1938), a.k.a. Typhoid Mary, who infected 22 people with typhoid fever. She is presumed to have infected 51 people, three of whom died, over the course of her career as a cook. She was twice forcibly isolated by public health authorities and died after a total of nearly three decades in isolation.

   Develop a mathematical model of an infectious disease that accounts for carriers. If you feel brave, try to analyze this model using Matlab to simulate the behavior of the solutions.