APMA: 0200
Homework #3

Due Date: October 2, 2015

1. Consider the the following differential equation:

$$\frac{dx}{dt} = \sin(x).$$

   (a) Find all fixed points for this equation.
   (b) At which points x does the flow have the greatest velocity to the right?
   (c) Find the acceleration $\frac{d^2x}{dt^2}$ as a function of x.
   (d) Find the points x where the flow has maximum positive acceleration.

2. For the following differential equation:

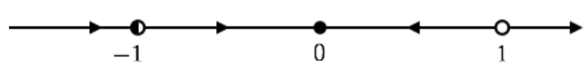
$$\frac{dx}{dt} = x - x^3$$

   sketch the vector field on the real line, find all fixed points, classify their stability, and sketch the graph of the solution curves $x(t)$ for different initial conditions.

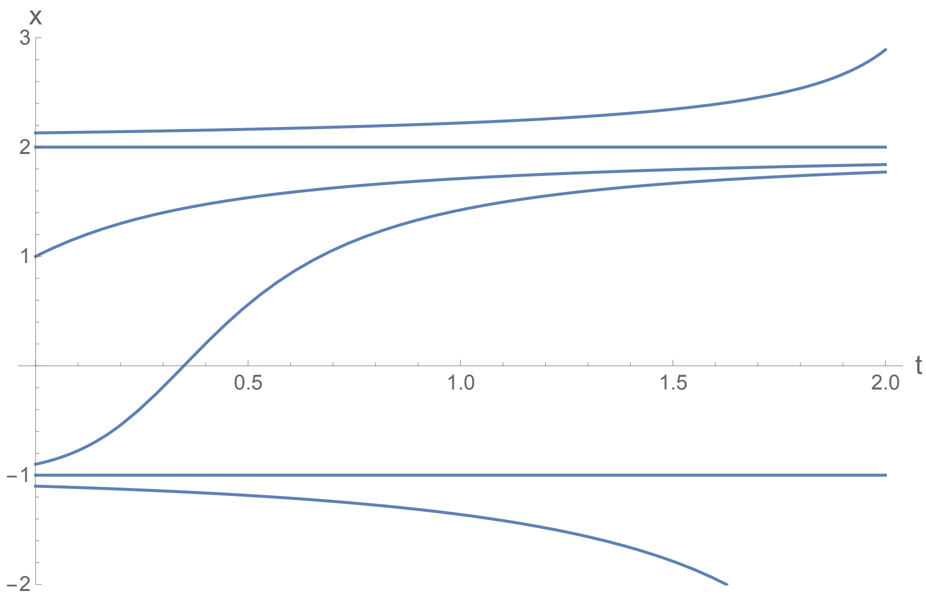
3. Given the differential equation

$$\frac{dx}{dt} = f(x),$$

   with corresponding phase portrait drawn below, find a formula for $f(x)$ that is consistent with it.



4. Find an equation $\frac{dx}{dt} = f(x)$ whose solutions are consistent with those shown below.



5. The growth of cancerous tumors can be modeled by the Gompertz law:

$$\frac{dN}{dt} = -aN\ln(bN),$$

where $N(t)$ is proportional to the number of cells in the tumor and $a, b > 0$ are parameters.

(a) Find all fixed points for this system.

(b) Sketch a graph of $f(N) = -aN\ln(bN)$.

(c) Interpret a and b biologically.

(d) Sketch the vector field and then graph the solutions curves $N(t)$ for various initial values.

6. For certain species of organisms, the effective growth rate $\frac{1}{N}\frac{dN}{dt}$ is highest for intermediate N. This is called the **Allee** effect. For example, imagine that it is too hard to find mates when N is very small, and there is too much competition for food and other resources when N is large.

(a) Show that $\frac{1}{N}\frac{dN}{dt} = r - a(N-b)^2$ provides an example of the Allee effect, if r, a, and b satisfy certain constraints, to be determined. (**Hint:** Graphing the function might help).

(b) Find all the fixed points for this system and analyze their stability.

(c) Sketch the solution curves $N(t)$ for different initial conditions and comment on how these curves compare to those found in class for the logistic equation. What are the qualitative differences if any.