APMA: 0200
Homework #2

Due Date: September 26, 2015

1. **Ricker Model** The so-called Ricker Model of salmon population growth is given by:

$$x_{n+1} = f(x_n) = x_n e^{r(1-x_n)},$$

where $r > 0$ is a growth rate.

(a) Find $\lim_{x\to\infty} f(x)$.

(b) For what values of x is f increasing? decreasing? At what value of x does $f(x)$ have an absolute minimum?

(c) For what values of x is f concave up? concave down? At what values of x does f have an inflection point.

(d) Using the results from items (a-c) sketch a graph of $f(x)$.

(e) Based on the graph of f, provide an explanation of why this could be a plausible model of population growth.

(f) Find any equilibrium points for this model and determine how their stability varies with r.

(g) Sketch cobweb diagrams for $x_0 = 1/2$ for the cases $r = 2$ and $r = 3$.

2. **Beaverton-Holt Model** Another model of salmon population growth is given by:

$$x_{n+1} = f(x_n) = \frac{rx}{1+x},$$

where $r > 0$ is a growth rate.

(a) Find $\lim_{x\to\infty} f(x)$.

(b) For what values of x is f increasing? decreasing? At what value of x does $f(x)$ have an absolute minimum?

(c) For what values of x is f concave up? concave down? At what values of x does f have an inflection point.

(d) Using the results from items (a-c) sketch a graph of $f(x)$.

(e) Based on the graph of f, provide an explanation of why this could be a plausible model of population growth.

(f) Find any equilibrium points for this model and determine how their stability varies with r.

(g) Sketch cobweb diagrams for $x_0 = 1$ for the cases $r = 2$ and $r = 4$.

3. **Shepherd Model** Our final model of salmon population growth is given by:

$$x_{n+1} = f(x_n) = \frac{rx}{1+x^2},$$

where $r > 0$ is a growth rate.

(a) Find $\lim_{x\to\infty} f(x)$.

(b) For what values of x is f increasing? decreasing? At what value of x does $f(x)$ have an absolute minimum?

(c) For what values of x is f concave up? concave down? At what values of x does f have an inflection point.

(d) Using the results from items (a-c) sketch a graph of $f(x)$.

(e) Based on the graph of f, provide an explanation of why this could be a plausible model of population growth.

(f) Find any equilibrium points for this model and determine how their stability varies with r.

(g) Sketch cobweb diagrams for $x_0 = 4$ for the cases $r = 2$ and $r = 10$.