Math 150 – FINAL EXAM – Spring 2006

[18] 1. Find the following limits. You must show all your work.

a) $\displaystyle\lim_{x\to 3} \frac{x^2-8x+15}{x^2+x-12}$

b) $\displaystyle\lim_{x\to 0} \frac{\tan 3x}{x}$

c) $\displaystyle\lim_{x\to\infty} (x-\sqrt{x^2-2x+5})$

[10] 2. USING THE DEFINITION of derivative find the derivative of $f(x)=\sqrt{x-2}$.

[30] 3. Find $f'(x)$ for the following functions. You do not need to simplify your answers.

a) $f(x)=e^x-\sqrt{x}+\cot x$

b) $f(x)=(x^2+5x+2)\cosh x$

c) $f(x)=\dfrac{1+\ln x}{x+\sin x}$

d) $f(x)=(x^2+4x-1)^{2/3}$

e) $f(x)=x^{\sin x}$

f) $f(x)=\displaystyle\int_1^x \frac{dt}{\sin^2 t+1}$

[8] 4. Let $y(x)$ be defined implicitly by $x^3+\ln(x+y)=y$. Find $y'(x)$.

[12] 5. Determine an equation of the tangent line to the curve $f(x)=\ln(x-1)$ at the point where the graph crosses the x-axis.

[10] 6. The combined electrical resistance R of R_1 and R_2, connected in parallel, is given by

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

where R, R_1 and R_2 are measured in ohms. R_1 and R_2 are increasing at rates of 1 and 1.5 ohms per second, respectively. At what rate is R changing when $R_1=50$ ohms and $R_2=75$ ohms?

[10] 7. Calculate the absolute maximum of $f(x)=\dfrac{x^2}{x^2+2}$ on $[-1,1]$.

[12] 8. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimizes the amount of material used.

[15] 9. Suppose that the DERIVATIVE, f', of a function f is given by $f'(x)=\dfrac{x}{x^2+9}$

a) Find the open intervals where f is increasing and those where f is decreasing.

b) Find the open intervals where f is concave up and those where f is concave down.

c) Suppose $f(0)=1$. Graph a plausible f using the above information.

[25] 10. Evaluate the following indefinite integrals.

a) $\int (\sqrt[3]{x} + \sqrt{x} + \operatorname{sech}^2 x)\, dx$

b) $\int (x^3 - 2)^2\, dx$

c) $\int xe^{x^2+4}\, dx$

d) $\int x\sqrt[3]{x+1}\, dx$

e) $\int (2 + \sin x)^3 \cos x\, dx$

[14] 11. Evaluate the following definite integrals.

a) $\int_0^{\sqrt{\pi/4}} x \sec x^2 \tan x^2\, dx$

b) $\int_0^3 f(x)\, dx$ where $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

[12] 12. Find the area of the region bounded by the curves $f = x^5$ and $g(x) = 4x^3$.

[14] 13. Let R be the region bounded by the curves $y = x^2 + 1,\ y = 0,\ x = 0$, and $x = 2$.

a) Set up, but DO NOT EVALUATE, the integral for the volume of the solid obtained by revolving this region around the $y-$axis.

b) Set up, but DO NOT EVALUATE, the integral for the volume of the solid obtained by revolving this region around the line $y = -1$.

[10] 14. The velocity function (in meters per second) for a particle moving along a line is $v(t) = -2t + 5$. Find the TOTAL DISTANCE traveled on the time interval [0,3].