Math 150

Final Exam

[25] 1. Find the following limits. You must show all your work.

a)
$$\lim_{x \to \infty} \frac{2 + x + 3x^2}{7x^2 + 6x - 1}$$

b) $\lim_{x \to 0} \frac{2x}{\sin 5x}$
d) $\lim_{x \to 4} \frac{\sqrt{x + 5} - 3}{x - 4}$
e) $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}$

c)
$$\lim_{x \to 4} \frac{x^2 - x - 12}{x^2 - 6x + 8}$$

[10] 2. Use the <u>definition</u> of the derivative to find f'(x) where $f(x) = \frac{1}{x-1}$.

[30] 3. Find $\frac{dy}{dr}$ for the following functions. You do not need to simplify your answer.

a)
$$y = 3x^2 - \frac{1}{\sqrt{x}} + \sin x - \cosh x$$

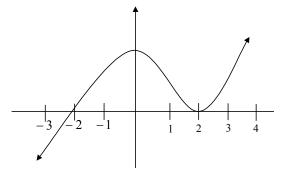
b) $y = (\ln(x^2 + 1))(\tan(2x + 3))$
c) $y = \cos^3(x + e^x)$
d) $y = \frac{x^2 + 3x + 5}{x^3 - x}$
e) $y = x^{\sin x}$
f) $y = \int_x^3 \frac{dt}{1 + t^4}$

[5] 4. Let y(x) be defined implicitly by $x^2y^2 - e^x + y = 4$. Find $\frac{dy}{dx}$.

[10] 5. Determine an equation of the tangent line to the curve of $f(x) = e^{2x}$ at the point where the curve crosses the line y = 1.

[10] 6. A ladder 15 ft. long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/min, how fast is the ladder sliding down the wall when the base of the ladder is 9 feet from the wall?

- [10] 7. A box with a square base and an open top has a surface area of 1875 cm² of material. Find the largest possible volume of the box.
- [10] 8. Find the absolute maximum and absolute minimum of $f(x) = x^3 12x + 5$ on [0, 3]. Give both x and y coordinates.
- [15] 9. Suppose that the DERIVATIVE of f' of a function f is given by



(This graph is not the graph of the function. It is the graph of the derivative of f).

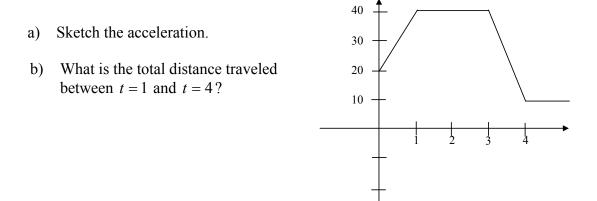
- a) Find the open intervals where f is increasing and those where f is decreasing.
- b) Find the open intervals where f is concave up and those where f is concave down.
- c) If f(0) = 0, draw a plausible graph of f.
- [25] 10. Evaluate the following indefinite integrals.
 - a) $\int \left(e^x + 3x \sinh x + \sqrt[3]{x}\right) dx$ b) $\int (x^2 + 1)^2 dx$ c) $\int \sec^2 x e^{\tan x} dx$ d) $\int \frac{x}{\sqrt{1+x}} dx$ e) $\int \frac{dx}{x \ln x}$

[15] 11. Evaluate the following definite integrals.

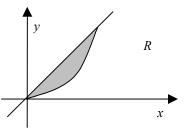
a)
$$\int_{0}^{4} \left(xe^{x^{2}}\right) dx$$
 b) $\int_{0}^{(\pi/4)^{2}} \frac{\sec^{2}\sqrt{x} dx}{\sqrt{x}}$ c) $\int_{-1}^{4} |x-1| dx$

[10] 12. Find the area of the region between the graphs of $y = x^2$ and y = -x.

[10] 13. Let the velocity of a particle be given by the following graph.



- [7] 14. Let R be the region bounded by the curves $y = \sqrt{x}$ and $y = \frac{1}{2}x$. Find the volume of the solid obtained by revolving the region around the *y*-axis.
- [8] 15. Let R be the region bounded by the curves $y = x^2$ and y = 3x.



- a) Set up, but DO NOT EVALUATE, the integral for the volume of the solid obtained by revolving this region around the x-axis.
- b) Set up, but DO NOT EVALUATE, the integral for the volume of the solid obtained by revolving this region around the line x = 3.