AM 165: Homework # 7 Solution part 2

- Suppose that X and Y has joint density

$$f(x,y) = \begin{cases} 2 & \text{if } x \geq 0, y \geq 0, x+y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the conditional expectation $E[Y|X]$.

Solution: First we need to compute $E[Y|X=x]$. To this end, we need to calculate $f(y|x)$. Note that the marginal density of X is, for an arbitrarily fixed $0 \leq x \leq 1$,

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy = \int_0^{1-x} 2dy = 2(1-x)$$

The conditional density is

$$f(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{2}{2(1-x)} = \frac{1}{1-x}$$

for all $0 \leq y \leq 1-x$, and $f(y|x) = 0$ otherwise. Therefore, for every $0 \leq x \leq 1$,

$$E[Y|X=x] = \int_{-\infty}^{\infty} yf(y|x)dy = \int_0^{1-x} y\frac{1}{1-x}\,dy = \frac{1}{1-x}\int_0^{1-x} ydy = \frac{1-x}{2}.$$

Thus, replace x by X we arrive at

$$E[Y|X] = \frac{1-X}{2}$$