Homework 3 Solution

3.4 Define events: A: value 1 fails
   B: value 2 fails
   C: value 3 fails

\[ P(Y=2) = P(A \cap B \cap C) = 0.8^2 = 0.512 \]
\[ P(Y=0) = P(A \cup B \cup C) = P(A) P(B) P(C) \]
\[ = 0.2(0.2)(0.2) = 0.072 \]
\[ P(Y=1) = 1 - 0.512 - 0.072 = 0.416 \]

\[ \text{by the law of total probability} \]

3.10 \[ E(Y) = \sum y p(y) = 1(0.4) + 2(0.3) + 3(0.2) + 4(0.1) = 2 \]
\[ E(Y) = \sum \frac{y}{n} p(y) = 1(0.4) + \frac{1}{2}(0.3) + \frac{1}{3}(0.2) + \frac{1}{4}(0.1) = 0.6417 \]
\[ E(Y^2) - 1 = E(Y^2) - 1 = [1(0.4) + 4(0.3) + 9(0.2) + 16(0.1)] - 1 = 5 - 1 = 4 \]
\[ V(Y) = E(Y^2) - (E(Y))^2 = 5 - 2^2 = 1 \]

3.12 In ex. 2.97, we've shown that \( P(Y=y) = \frac{1}{n} \), for \( y = 1, \ldots, n \).

So, \[ E(Y) = \sum_{y=1}^{n} \frac{y}{n} = \frac{1}{n} \left( 1 + 2 + \cdots + n \right) = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2} \]
\[ E(Y^2) = \sum_{y=1}^{n} \frac{y^2}{n} = \frac{1}{n} \left( 1^2 + 2^2 + \cdots + n^2 \right) = \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6} \]
\[ V(Y) = E(Y^2) - (E(Y))^2 \]
\[ = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{n^2 - 1}{12} \]

3.7 Define \( G \) = gain in drawing one card

<table>
<thead>
<tr>
<th>( G )</th>
<th>( p(G) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>( \frac{2}{15} )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{1}{15} )</td>
</tr>
<tr>
<td>-4</td>
<td>( \frac{1}{15} )</td>
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\[ E(G) = \sum G p(G) = 15 \left( \frac{2}{15} \right) + 5 \left( \frac{1}{15} \right) - 4 \left( \frac{1}{15} \right) = \frac{16}{15} = 0.31 \]

The expected gain is \( \$0.31 \).
3.30 \( Y = \# \) of falsified application forms \( \sim \) Binomial \((5, .35)\)

\[
P(Y \geq 1) = 1 - P(Y = 0) = 1 - (\binom{5}{0} .35^0 .65^5) = 1 - .116 = .884
\]

\[
P(Y \geq 2) = 1 - P(Y = 0) - P(Y = 1) = 1 - .116 - (\binom{5}{1} .35^1 .65^4) = .884 - .312 = .572
\]

3.36 \( Y = \# \) of sets out of 5 that detect the missile \( \sim \) Binomial \((5, .9)\)

\[
P(Y = 4) = \binom{5}{4} .9^4 .1 = .3888
\]

\[
P(Y \geq 1) = 1 - P(Y = 0) = 1 - (\binom{5}{0} .9^0 .1^5) = .99999
\]

b. of radar sets, the probability of at least one detection is

\[
P(Y \geq 1) = 1 - P(Y = 0) = 1 - (\binom{n}{0} .9^0 .1^n) = 1 - (\cdot 1)^n
\]

we know \( 1 - (\cdot 1)^n = .999 \Rightarrow n = 3 \)

3.40 \( Y \sim \) Binomial \((10, .1)\)

\[
E(Y) = np = 10 \times .1 = 1
\]

\[
V(Y) = npq = 10 \times .1 \times .9 = .9
\]

3.45 \( Y = \# \) of defective motors out of 10. Then \( Y \sim \) Binomial \((10, .08)\)

\[
\therefore E(Y) = (\cdot 08)(10) = .8
\]

The seller gains \$1,000 on the sale of 10 motors & loses \$200 for each defective. The seller's expected gain is

\[
1000 - 200E(Y) = 1000 - 200(1.8) = (\frac{1}{4})840
\]