Homework 3 Solution

3.4 Define events: A: value 1 fails, B: 2 fails, C: 3 fails (independent)

$P(Y=2) = P(\bar{A} \cap \bar{B} \cap \bar{C}) = 0.8^3 = 0.512$

$P(Y=0) = P(A \cap (B \cup C)) = P(A)\,P(B \cup C)$

$= .2(.2 + .2 - .2^2) = .072$

$P(Y=1) = 1 - .512 - .072 = .416$

↑ by the law of total probability

3.10 $E(Y) = \sum y\,p(y) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = 2$

$E(Y) = \sum \frac{1}{y} p(y) = 1(.4) + \frac{1}{2}(.3) + \frac{1}{3}(.2) + \frac{1}{4}(.1) = .6417$

$E(Y^2 - 1) = E(Y^2) - 1 = [1(.4) + 4(.3) + 9(.2) + 16(.1)] - 1 = 5 - 1 = 4$

$V(Y) = E(Y^2) - (EY)^2 = 5 - 2^2 = 1$

3.12 In ex. 2.97, we've shown that $P(Y=y) = \frac{1}{n}$, for $y = 1, \cdots, n$.

So, $E(Y) = \sum_{y=1}^{n} \frac{y}{n} = \frac{1}{n}(1 + 2 + \cdots + n) = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2}$

$E(Y^2) = \sum_{y=1}^{n} \frac{y^2}{n} = \frac{1}{n}(1^2 + 2^2 + \cdots + n^2) = \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}$

$V(Y) = E(Y^2) - (EY)^2$

$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{n^2 - 1}{12}$

3.17 Define G = gain in drawing one card

G	$p(G)$
15	$\frac{2}{13}$
5	$\frac{2}{13}$
-4	$\frac{9}{13}$

$E(G) = \sum G\,p(G) = 15\left(\frac{2}{13}\right) + 5\left(\frac{2}{13}\right) - 4\left(\frac{9}{13}\right) = \frac{4}{13} = .31$

The expected gain is \$.31.

3.30 $Y = \#$ of falsified application forms $\sim$ Binomial $(5, .35)$

$P(Y \geq 1) = 1 - P(Y=0) = 1 - \binom{5}{0}(.35)^0(.65)^5 = 1 - .116 = .884$

$P(Y \geq 2) = 1 - P(Y=0) - P(Y=1)$

$= 1 - .116 - \binom{5}{1}(.35)^1(.65)^4$

$= .884 - .312 = .572$

3.36 a. $Y = \#$ of sets out of 5 that detect the missile $\sim$ Binomial $(5, .9)$

$P(Y=4) = \binom{5}{4}(.9)^4(.1) = .32805$

$P(Y \geq 1) = 1 - P(Y=0) = 1 - \binom{5}{0}(.9)^0(.1)^5 = .99999$

b. n radar sets, the probability of at least one detection is $P(Y \geq 1) = 1 - P(Y=0) = 1 - \binom{n}{0}(.9)^0(.1)^n$

$= 1 - (.1)^n$

we know $1 - (.1)^n = .999 \Rightarrow n = 3$

3.40 Y = # of success

$Y \sim$ Binomial $(10, .1)$

$E(Y) = np = 10 \times (.1) = 1$

$V(Y) = npq = 10(.1)(.9) = .9$

3.43 $Y = \#$ of defective motors out of 10. Then $Y \sim$ Binomial $(10, .08)$.

$\therefore E(Y) = (.08)(10) = .8$

The seller gains \$1,000 on the sale of 10 motors & loses \$200 for each defective. The seller's expected gain is

$1000 - 200E(Y) = 1000 - 200(.8) = (\$)\ 840$