Solution for HW 2

2.57 $P(A) = 0.5,\ P(B) = 0.3,\ P(A\cap B) = 0.1$

a. $P(A|B) = P(A\cap B)/P(B) = \frac{.1}{.3} = \frac{1}{3}$

b. $P(B|A) = P(A\cap B)/P(A) = .1/.5 = .2$

c. $P(A|A\cup B) = P(A\cap(A\cup B))/P(A\cup B)$

additive law of probability $\rightarrow$ $= \frac{P(A)}{P(A)+P(B)-P(A\cap B)} = \frac{.5}{.5+.3-.1} = \frac{5}{7}$

d $P(A|A\cap B) = P(A\cap(A\cap B))/P(A\cap B) = 1$

e. $P(A\cap B|A\cup B) = P((A\cap B)\cap(A\cup B))/P(A\cup B) = \frac{P(A\cap B)}{P(A\cup B)} = \frac{1}{7}$

2.58 The necessary probabilities can be obtained directly from the table. To determine whether or not A & M are independent, look at

$P(A) = .6$ & $P(A|M) = P(A\cap M)/P(M) = \frac{.24}{.4} = .6$

$\Rightarrow$ A & M are independent.

For $\bar{A}$ & F, look at $P(\bar{A}) = .4$ & $P(\bar{A}|F) = \frac{P(\bar{A}\cap F)}{P(F)} = \frac{.24}{.6} = .4$

$\Rightarrow$ $\bar{A}$ & F are independent.

2.59 Let r denote recessive white & R denote dominant red, then

a. $P(\text{at least one } R) = P(\text{Red}) = \frac{3}{4}$

b. $P(\text{at least one } r) = P(rr \text{ or } rR \text{ or } Rr) = \frac{3}{4}$

c. $P(\text{one } r|\text{Red}) = \frac{0.5}{0.75} = \frac{2}{3}$

2.67 ① In order to show that A & $\bar{B}$ are independent, it is necessary to show that $P(A \cap \bar{B}) = P(A)P(\bar{B})$

$$P(A) = P(A \cap S) = P(A \cap (B \cup \bar{B}))$$

$$= P((A \cap B) \cup (A \cap \bar{B}))$$

additive law

$(A \cap B) \cap (A \cap \bar{B}) = \phi \longrightarrow \;= P(A \cap B) + P(A \cap \bar{B})$

independence of A & B $\longrightarrow \;= P(A)P(B) + P(A \cap \bar{B})$

$$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A)P(B) = P(A)(1 - P(B))$$

$$= P(A)P(\bar{B})$$

② A & $\bar{B}$ are independent $\Leftrightarrow$ $\bar{B}$ & A are independent

$\Rightarrow$ $\bar{B}$ & $\bar{A}$ are independent (use the conclusion in ①)

2.69 Let W denote the event that she wins & assume that she begins with her right hand. Note there are exactly 3 ways to win once the choice of hand has been made. Let H denote a hit & M denote a miss there are HHH, HHM & MHH. Then

$$P(W|R) = P(HHH|R) + P(HHM|R) + P(MHH|R)$$

$$= (.7)(.4)(.7) + (.7)(.4)(.3) + (.3)(.4)(.7)$$

$$= 0.364$$

2.72 $P(A) = .5$, $P(B) = .2$, A & B independent $\Rightarrow P(A \cap B) = P(A)P(B)$

$= .1$

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .5 + .2 - .1 = .6$

b. $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - .6 = .4$

c. $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B) = 1 - .1 = .9$

2.97 a. Obviously, it's $\frac{1}{n}$

b. $P(2^{nd}\text{ correct}) = P(2^{nd}\text{ correct } \& \ 1^{st}\text{ false})$

$= P(2^{nd}\text{ correct} \mid 1^{st}\text{ false})\, P(1^{st}\text{ false})$

$= \frac{1}{n-1} \cdot (1 - \frac{1}{n}) = \frac{1}{n}$

$P(3^{rd}\text{ correct}) = P(3^{rd}\text{ correct } \& \ 2^{nd}\text{ false } \& \ 1^{st}\text{ false})$

$= P(3^{rd}\text{ correct} \mid 2^{nd}\text{ false } \& \ 1^{st}\text{ false})$

$\times P(2^{nd}\text{ false} \mid 1^{st}\text{ false}) \times P(1^{st}\text{ false})$

$= \frac{1}{n-2} \cdot \frac{n-2}{n-1} \cdot \frac{n-1}{n} = \frac{1}{n}$

c. $P(\text{gain access}) = P(\text{first try correct}) + P(\text{second correct})$

$+ P(\text{third correct})$

$= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{3}{7}$

2.103 Define P: positive response

M: respondent was male

F: respondent was female

Then $P(P|F) = .7$, $P(P|M) = .4$, $P(M) = \frac{1}{4}$

Use Bayes' rule, $P(M|\bar{P}) = \dfrac{P(\bar{P}|M)P(M)}{P(\bar{P}|M)P(M) + P(\bar{P}|F)P(F)}$

$= \dfrac{(.6)(\frac{1}{4})}{(.6)(\frac{1}{4}) + (.3)(\frac{3}{4})} = .4$

2.111 Define G: student guess

C: student correctly answers question

Then $P(G) = .2$, $P(C|\bar{G}) = 1$ & $P(C|G) = .25$

$\Rightarrow P(\bar{G}|C) = \dfrac{P(C|\bar{G})P(\bar{G})}{P(C|\bar{G})P(\bar{G}) + P(C|G)P(G)} = \dfrac{(1)(.8)}{(1)(.8) + (.25)(.2)}$

$= \frac{.8}{.85} = .9412$

2.120 Sample space $S = \{AA, AB, AC, AD,$
$BA, BB, BC, BD,$
$CA, CB, CC, CD,$
$DA, DB, DC, DD\}$

positions that the spinner lands on (distinguish different order)

It's not difficult to discover the only possible value for Y is 2 and 3.

$P(Y=2) = \binom{4}{2}\left(\frac{1}{16}\right) = \frac{3}{4}$

$P(Y=3) = \binom{4}{3}\left(\frac{1}{16}\right) = \frac{1}{4}$