

APMA 1200: Operational Analysis: Probabilistic Models 2007

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Office hour: Wednesday and Friday 11:00-12:00.

Prerequisite. APMA 1650 or MATH 1610 or equivalent. The students are required to have a solid understanding of basic probability and calculus.

Course description. This course serves as an introduction to stochastic processes and stochastic optimization. It will cover basic stochastic processes such as simple random walk, Markov chains, Poisson processes, and Brownian motion. It will also offer introductory level treatment to stochastic optimization problems, such as optimal stopping and optimal control, based on dynamic programming principle.

Connection to APMA 1210. Students often think *APMA 1210: Operational Analysis: Deterministic Models* is the follow-up or precursor of this course. The fact is that there is little connection between these two courses. Students can take these two courses independently.

Textbook: No textbook required. Students can download the lecture notes from the course homepage during the semester. Below is a list of reference books. The students are not required to read these reference books – they are listed just for your information.

Stochastic Processes.

- Karlin, S. and Taylor, H.M. 1984. *An Introduction to Stochastic Modeling*. Academic Press, San Diego, CA, USA. [elementary]
- Karlin, S. and Taylor, H.M. 1975. *A First Course in Stochastic Processes*. Academic Press, New York, USA. [intermediate]
- Karlin, S. and Taylor, H.M. 1975. *A Second Course in Stochastic Processes*. Academic Press, New York, USA. [intermediate to advanced]
- Ross, S.M. 1983. *Stochastic Processes*. John Wiley & Sons, New York, USA. [intermediate]
- Resnick, S.I. 1992. *Adventures in Stochastic Processes*. Birkhäuser, Boston, MA, USA. [intermediate]
- Feller, W. 1970. *An Introduction to Probability Theory and Its Applications*. Vol 1. John Wiley and Sons, New York, USA [Intermediate]

Stochastic Optimization.

- Bertsekas, D.P. 1987. *Dynamic Programming: Deterministic and Stochastic Models*. Prentice Hall, Englewood Cliffs, NJ, USA.
- Bertsekas, D.P. 1995. *Dynamic Programming and Optimal Control*. Athena Scientific, Belmont, MA, USA.
- Ross, S.M. 1983. *Introduction to Stochastic Dynamic Programming*. Academic Press, San Diego, CA, USA.

Homework: Homework is an integral part of the course and is mandatory. Late homework will only be accepted if an adequate reason is given the day before homework is due.

Grades: There will be one in-class midterm exam, one take-home final exam, and (probably) a computational project. Overall grade will be given based on:

1. If there is a computational project: homework (20%), midterm (20%), final exam (40%), and computational project (20%).
2. If there is no computational project: homework (25%), midterm (25%), final exam (50%).

Syllabus:

- Review of basic probability theory:
 1. Sample space, probability, events, random variables.
 2. Basic probability rules, expectation, variance and covariance.
 3. Discrete and continuous distributions, conditional distribution, conditional expectation.
 4. Independence, law of total probability, tower property.
- Simple random walk.
 1. Definitions, hitting times, and gambler's ruin problem.
 2. First step analysis.
 3. Recurrence and transience.
 4. Wald's identity.
 5. Combinatorics, reflection principle.
- Markov chains.
 1. Definitions, transition probability matrix, Chapman-Kolmogorov Equation.
 2. Hitting times, strong Markov property.
 3. Classification of states, recurrence and transience, null recurrent and positive recurrent

4. Stationary distribution, and convergence to stationary distribution, coupling argument.
 5. Branching processes.
- Martingales.
 1. Definitions.
 2. Examples of classical martingales.
 3. Optional sampling theorem and its application.
 - Poisson process.
 1. Definitions and basic properties.
 2. Interarrival times and waiting times.
 3. Inspector's paradox.
 4. Poisson process with classified events.
 - Brownian motion.
 1. Definitions and basic properties.
 2. Reflection principle, hitting times, running maxima.
 3. Basic martingales associated with Brownian motion and their applications.
 4. Various topics Brownian motion.
 - Stochastic optimization and its applications.
 1. Optimal Stopping.
 2. Optimal control.
 3. Dynamic programming principle, Hamilton-Jacobi-Bellman (HJB) Equation.
 4. Verification argument.
 5. Applications.