New singular solutions of the biharmonic NLS

Guy Baruch,
Tel aviv University

We consider blowup-type singular solutions of the fourth-order (biharmonic) nonlinear Schrodinger (BNLS)

$$i\psi_t\left(t,\vec{x}\right) - \Delta_{\vec{x}}^2\psi + \left|\psi\right|^{2\sigma}\psi = 0, \qquad \vec{x} \in \mathbb{R}^d.$$

Our formal and informal analysis, and numerical evidence, indicate that the blowup formation greatly resembles that of the standard (harmonic) NLS. However, the lack of some key analytic tools still makes BNLS theory challenging.

In the L^2-critical case $\sigma \cdot d = 4$, we rigorously prove that the collapsing core of the solution converges to a self-similar profile, and bound the blowup rate.

In the supercritical case $\sigma \cdot d > 4$, we use asymptotic analysis to find and characterize new peak-type and ring-type singular solutions of the BNLS, which also converge to a self-similar profile. We determine whether the ring-type solutions blowup at either a sphere ("standing ring") or a point ("shrinking ring"), as determined by σ and d. These findings are verified numerically, using an adaptive mesh, at focusing factors of up to 10^8.

Joint work with Gadi Fibich and Elad Mandelbaum.