The flow near the ground state for the mass critical gKdV equation

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I will consider the mass critical one dimensional (gKdV) problem

$$\partial_t u + (u_{xx} + u^5)_x = 0, \quad (t,x) \in \mathbb{R} \times \mathbb{R}.$$

This problem belongs to a class of canonical focusing dispersive nonlinear problems which appear in various physical situations, in particular the cubic two dimensional nonlinear Schrödinger equation from nonlinear optics. (gKdV) admits well known ground states traveling waves solutions $u(t,x) = Q(x-t)$ which are unstable. We will in this talk describe completely the instability of these exceptional solutions and show that any perturbation in a suitable class leads to one of the following three scenario: (Blow up) in a universal regime which is fully described, (Soliton) where the solution asymptotically converges to another solitary wave, (Exit) where the solution moves away from the manifold of solitary waves in the scaling invariant L^2 space. The first and third regimes are moreover stable by perturbation of the data. Finally, I will prove the existence and uniqueness of a minimal blow up element, and show how the long time dynamics of this object are directly related to the the long time dynamics of the solution in the (Exit) regime, and the associated scattering conjecture.

This is joint work with Y. Martel (Versailles) and F. Merle (Cergy and IHES).