**APMA 0410**

**Final Exam 2012**

**Problem 1.** Do a qualitative analysis of the following system of differential equations:

$$\left[\begin{matrix}\frac{dx}{dt}=x-3y\\\frac{dy}{dt}=2x-3y.\end{matrix}\right.$$

There is no need to do the eigenvalue-eigenvector analysis, only what it takes to draw a qualitatively accurate phase portrait.

**Problem 2.** Do the complete analysis, including eigenvalue-eigenvector analysis and drawing of separatrices, of the following system of differential equations:

$$\left[\begin{matrix}\frac{dx}{dt}=x+y\\\frac{dy}{dt}=2y.\end{matrix}\right.$$

Find *x* and *y* at time *t* = ln(2) given that *x*(0) = 2 and *y*(0) = 1.

**Problem 3.** Solve the Initial-Value Problem:

$$\left[\begin{matrix}\frac{dx}{dt}=x^{3}\\x\left(0\right)=1.\end{matrix}\right.$$

What is the time interval over which *x*(*t*) is defined?

**Problem 4.** Find the general solution of the differential equation:

$$\frac{dV}{dt}=-2V+te^{-t}$$

Note that this equation (albeit with different coefficients) describes the membrane potential of a neuron driven by an EPSP of shape $te^{-t}$.

Find *V* at time *t* = 1, given the initial condition: *V*(0) = 1. (Do not compute expressions such as 1/*e*. Leave them in your final result.)

**Problem 5.** Do a complete analysis of the following system of differential equations:

Characterize the shapes and slopes of all trajectories as *t* goes to –∞ and to +∞. Draw the separatrices and characterize the basins of attraction.

**Bonus Problem.** Do a complete analysis of the following system of differential equations:

Describe the trajectories as *t* goes to –∞ and to +∞. Draw the separatrices and describe the basins of attraction.