**APMA 0410 MIDTERM EXAM**

**October 14, 2012**

Three Poisson neurons, A, B and C, fire independently of each other. Their rates are, respectively, 3 spikes/s, 5 spikes/s, and 9 spikes/s.

**Problem 1.**

We denote the numbers of spikes of A, B and C during a given 5-second period by *X*A, *X*B and *X*C. Compute the expected values and variances of the following three random variables:

*Y*1 = 2*X*A; *Y*2 = *X*A + *X*B + *X*C; *Y*3 = 2*X*A – 3*X*B + *X*C.

**Problem 2.**

During the period of time 1 ms to 1000 ms, it is observed that the last firing of neuron B occurred at time *t* = 900 ms. Given that observation, what is the conditional probability that the next spike of B will occur after time *t* = 1200 ms?

**Problem 3.**

When neuron C is not stimulated, it fires at 9 spikes/s as mentioned above. However, when stimulated, it fires at some higher rate. Find a number *r* such that if neuron C is observed to fire more than *r* spikes during a given period of 9 seconds you would conclude at a significance level of 0.025 that it was stimulated during that period. Explain your answer.

**Problem 4.**

Neurons D, E and F have refractory periods such that, during a given short enough period of time *T*, they can fire at most one spike each. D and E fire independently of each other. The probability that D fires during *T* is 0.3 and the probability that E fires during *T* is 0.5. Neuron F receives an excitatory synapse from neuron D and an inhibitory synapse from neuron E. As a result, the probability that neuron F will fire during *T* is:

0.3 if both D and E fired during *T*,

0.1 if E but not D fired during *T*,

0.8 if D but not E fired during *T*,

0.5 if neither D nor E fired during *T*.

Given that F was observed to fire during *T*, what is the probability that D fired during *T*?

**Problem 5.**

We record simultaneously from 100 motor cortical neurons for a period *T* of length 10 ms, under one of two conditions: Rest (R) or Movement (M). In each of the two conditions, all neurons are Poisson and independent. Under condition R, all neurons fire at 5 spikes/s. Under condition M, all neurons fire at 100 spikes/s.

We denote by *Ui* the number of spikes of neuron *i* for any trial such that the condition during *T* was R, and by *Vi*  the number of spikes of neuron *i* for any trial such that the condition during *T* was M. We also denote:

*U* = *U*1 + *U*2 +… + *U*100

*V* = *V*1 + *V*2 +… + *V*100.

Is the distribution of *U* binomial? Is it approximately Poisson? Is it approximately normal? Specify the parameter(s) of the distribution for each positive answer.

Is the distribution of *V* binomial? Is it approximately Poisson? Is it approximately normal? Specify the parameter(s) of the distribution for each positive answer.

**Bonus problem.**

We denote by *Wi* the number of spikes of neuron *i* during *T* on trials where we don’t observe the condition but are told that R and M occur with probability 0.5 each. We also denote:

*W* = *W*1 + *W*2 +… + *W*100

Are *W*1 and *W*2 independent?

Sketch the distribution of *W*. Is it binomial, approximately Poisson, approximately normal?