What does it mean to be educated ?

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"Education is Mark Hopkins on a log and a student on the other end." -- Pres. James Garfield.

"The goal of education is to elicit more of the same." -- Hadassah F. Davis

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This essay was occasioned by the remark of a friend, a physician turned biomedical researcher, who told me that most doctors are uneducated. Was this the statement of a snob ? No? Yes ? Maybe ?

I don't know about you, Dear Reader, but I am an educated man. I know this for a certainty because in late June, 1943, at the Harvard Graduation Ceremony, Pres. James Bryant Conant welcomed me and several hundred others into "The Fellowship of Educated Men."

What did I have to accomplish to merit such admission? Well, I had to pass a swimming test. Then, towards the last semester of my senior year I had to scramble around and find a language with which I could satisfy the language requirement. Luckily I thought of German, and presto chango, German helped me become an educated man. Had I had any courses in English Literature ? No. Had I had any courses in philosophy or on history or on art ? No. But I had passed the criteria of the hour and the place and I was declared educated. We know when a cake sitting in the oven is half baked. In retrospect, I think that by late June 1943, according to certain criteria, I was only half or perhaps one tenth baked --education wise. Education is an endless process.

I often think, in retrospect, that my undergraduate education came as much from the classmates I hung out with as from the specific courses I took.

Prior to graduation from college, a student must be admitted to the college (to be educated !) . In the good old days of Presidents John Adams and Thomas Jefferson, in order to get into a college you had to know more than a bit of Latin and Greek. I got into college having had only two years of Latin, two years of French , two years of German, and more than a bit of mathematics. I've heard that these days, perhaps in jest, the only admission requirement is that the young student have a picture on Facebook and comes forward there with a firm philosophy of life. The Admissions Office at Brown supplies a very fuzzy statement of what is required for admission and owns up to unpublicized objective as well as subjective criteria as to which applicants will get the nod.

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I now take up a very specific field -- mathematics -- with considerable trepidation. In his autobiography , Mark Kac said that "Education is what remains after you've forgotten all you learned in school."

I now raise the question "What should an educated mathematician (EM) know ?"

Should a EM know what a group is ? A coset ? What an algebraic variety is? What the Gaussian abscissas are ? Should an EM have a personal opinion as to whether Cantor's set theory should be accepted or rejected ? Should an EM have sound opinions as to the current value of prostaphaeresis ? Should he/she have read through the proof of Fermat's last theorem or even read the proof that π (pi) is transcendental ? Should an EM know by heart the law of sines for a tetrahedron ? Can an EM provide an axiomatic definition of the real number system ? Can an EM provide the arrows for some well known category ? Should an EM have an idea as to what Shannon entropy says ? Should he/she know how to algorithmize the selection of a random number or the numerical solution of a hyperbolic partial differential equation ?   Should an EM be able to teach a course on combinatorial topology ? Can an EM expatiate on the semantics and semiotics of a formula ?

Has an EM formulated a value judgement as to whether a specific piece of mathematics is deep or is trivial?[1] Can an EM distinguish a functor from a function? Is Koornwinder's product formula on the tip of every EM's tongue ? Has the Jordan canonical form of a matrix become second nature ?

And so on and so on well into the night. The answers are yes, no, maybe, sort of, and it depends.

The field of mathematics is now so vast that it defies the ability of a person to know it all or even to know what there is to know The change came shortly after the First World War. Alexander Ostrowski told me that when he came up for his doctorate at Göttingen in 1920, he was expected to know all of the important mathematics Ostrowski may have been bragging or joking , but the date strikes me about right as to when the mathematical community became a community of specialists. A story has it that when John von Neumann was asked how much mathematics he knew, he answered "Twenty eight percent."

The age of mathematical specialism has been characterized and lamented by David Mumford who wrote

"The thing that leaps to mind is something about the suicidal tendency in math to get more and more technical and never to think about explaining one's ideas to mathematicians in other fields of math (let alone other scientists or even the general public). The field has a strange psychology linked to the fear of being thought dumb if you don't know everything."

Does severe specialism mean that the vaunted unity of mathematics is now threatened ? This is quite possible. Those who believe it is not so threatened support their opinion by saying that a specialist in one field can easily learn what is going on in another field

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[1] Dame Mary Cartwright, an eminent British mathematician, told me that when she came up for admission to an academic program, she was expected to know two different proofs of the "Nine point circle theorem." She may have been satirizing what, in fact, she was expected to know.

because the "symbols are out there for everyone to read." I reject this because I do not believe that mathematical practice and applicability, that mathematical understanding, knowledge, insight, aesthetic or other values can be based only on familiarity with a set of symbols. These symbols reside in the mathematical experience of the world mathematical community and they acquire their meaning from the communications and discussions within this community. Isolated, uncommunicated mathematics has no existential meaning.

I return to the question "What should an educated mathematician know ? " *Group I* :These are the courses that are required for a bachelor's degree in Mathematics. They represent the current wisdom of the mathematical and scientific community. This group can be symbolized by what Mark Hopkins on a log teaches a student.   *Group II* : More of the same and more of the same. And yet this is hardly sufficient :

> "The antithesis between a technical and a liberal education is fallacious. There can be no adequate technical education which is not liberal, and no liberal education which is not technical."   - Alfred North Whitehead

> Education is an endless process.