Chapter 3. Failure of the Quants

Abstract:

The Quants made several interrelated errors. They based the expected house price upon the current price and drift, and assumed that the distribution of capital gains is unchanging. The Quants ignored publicly available information in forming expectations. The ultimate determinant of the values of CDO's was the ability of the mortgagors to service their debts. It was public information that the mortgages were of most dubious value, no due diligence was performed in the ratings and the debt/income of the mortgagors was rising. Hence it was improbable that the distribution of house prices would remain constant. Systemic risk was ignored by the Quants who just focused upon current prices. Ratings of the tranches were not based upon the quality of the underlying mortgages. They were all in the same bundle. The rating depended upon who got paid first in the stack of loans. The key question was how to rate and price the tranches. The issue concerned the correlation of the tranches. If a pool of loans started experiencing difficulties, and a certain percent of them defaulted, what would be the impact upon each tranche? The "apples in the basket model" made one prediction. Another very different one is "the slice of bread in the loaf" model.

This chapter considers the role of the "Quants" in the crisis. The financial system is an intermediary between savers and investors. Mathematics is essential to price securities traded by savers, intermediaries and investors, and to adjust risk to the preferences of savers and investors. Assets are priced according to the principle that the price should be equal to the expectation of the present value of the future income. The future income stream and time profile of future interest rates are crucial variables, but they are stochastic and unknown when the pricing decision is made. Similarly the pricing of longer-term bonds depends upon the expectations of future short term interest rates. This pricing is based upon the term structure hypothesis that the long rate is a geometric average of future short rates.

When the Federal Reserve changed its operating policy in 1979, interest rates became more responsive to fluctuations in aggregate demand and supply shocks. Figure 3.1 graphs the 10-year Treasury Constant Maturity Rate and figure 3.2 graphs the 3-month Treasury bill rate. The variability of the Treasury bond rate carried over to the mortgage market is graphed in figure 3.3.

As a result of the considerably increased volatility of both short and long term interest rates there was an increased the demand for risk management. One took a great speculative risk if a bond position was not hedged.

In the volatile economic climate of the 1970s, trading in financial futures was introduced. Between 1970 and 1984 futures trading volume increased tenfold and by the end of 1984 financial futures accounted for 50% of total futures trading. The Federal Reserve Board (FRB), Securities Exchange Commission (SEC) and Commodities Futures Trading Commission (CFTC) were directed by an act of Congress in 1982 to carry out a joint study of the economic purposes of futures and options markets, to consider their effects upon capital formation, and to evaluate the adequacy of regulation. I was commissioned to write on the first topic which later resulted in my book *The Economics of Futures Markets (1986)*. My work seemed to satisfy the FRB, SEC and CFTC that these markets served useful economic functions and were not “gambling casinos”.

The interest rate volatility (figures 3.1- 3.3) above generated a demand by Wall Street for Quants. The Quants are a group of physicists, mathematicians and computer science experts who practice “financial engineering”. They develop and apply models to devise derivatives that would permit the risks to be hedged to an optimal extent, making the new securities attractive to both savers and investors. The channeling saving to investment is important for the growth process.

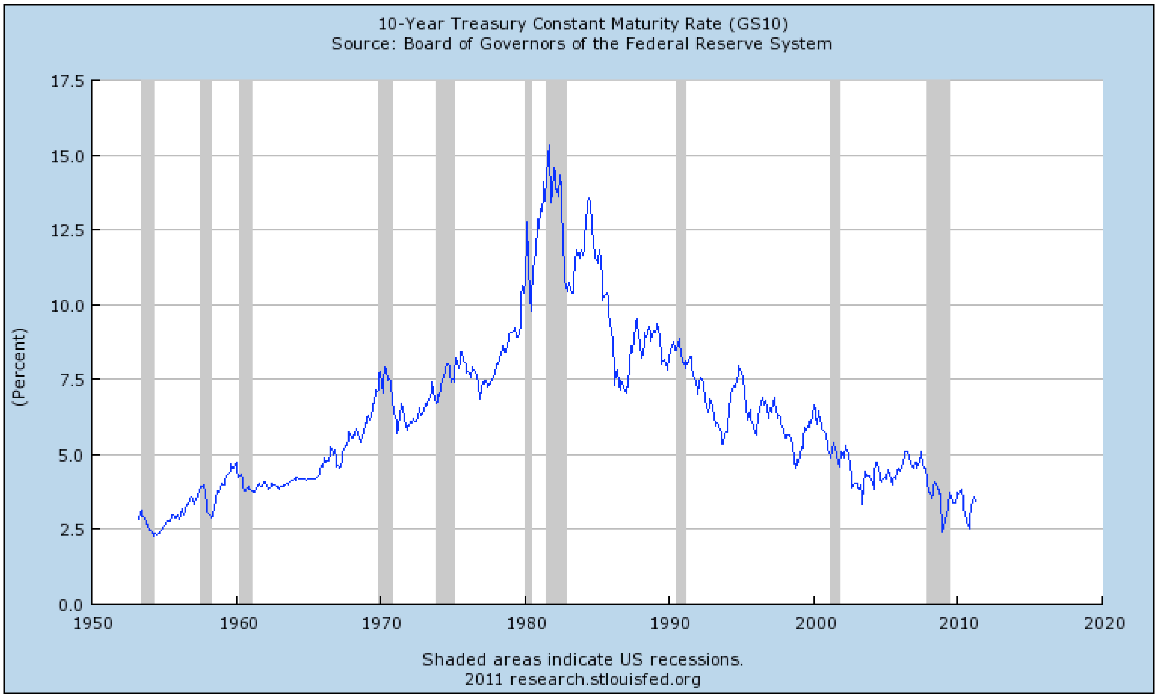

Figure 3.1 10-year Treasury Constant Maturity Rate.

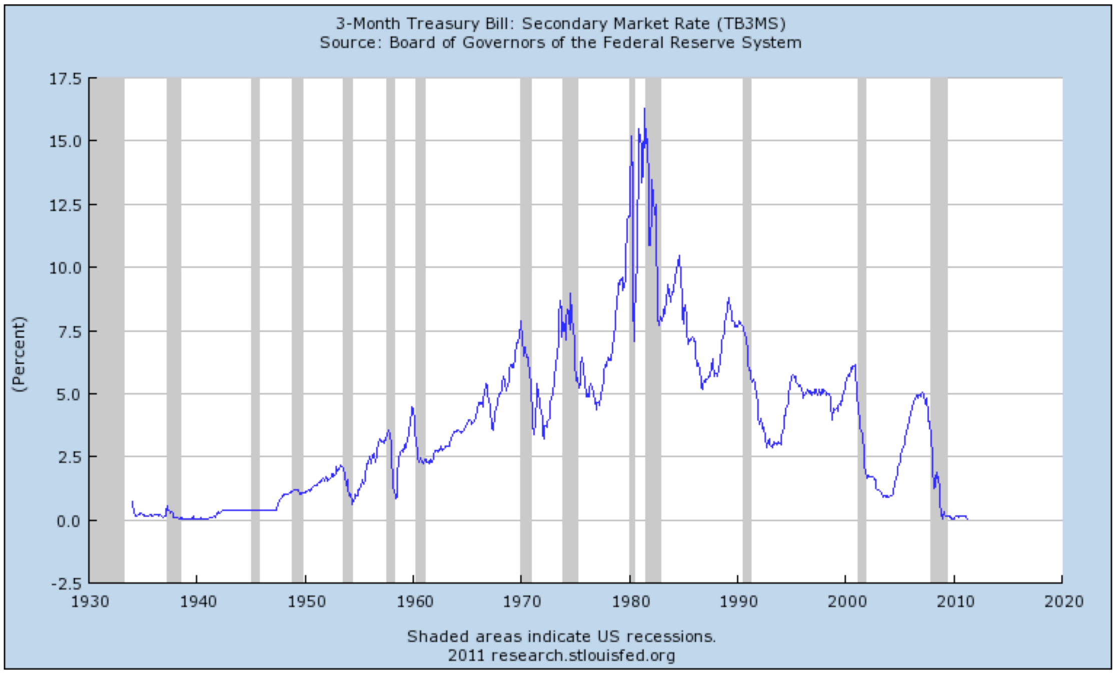

Figure 3.2 3-month Treasury Bill Rate

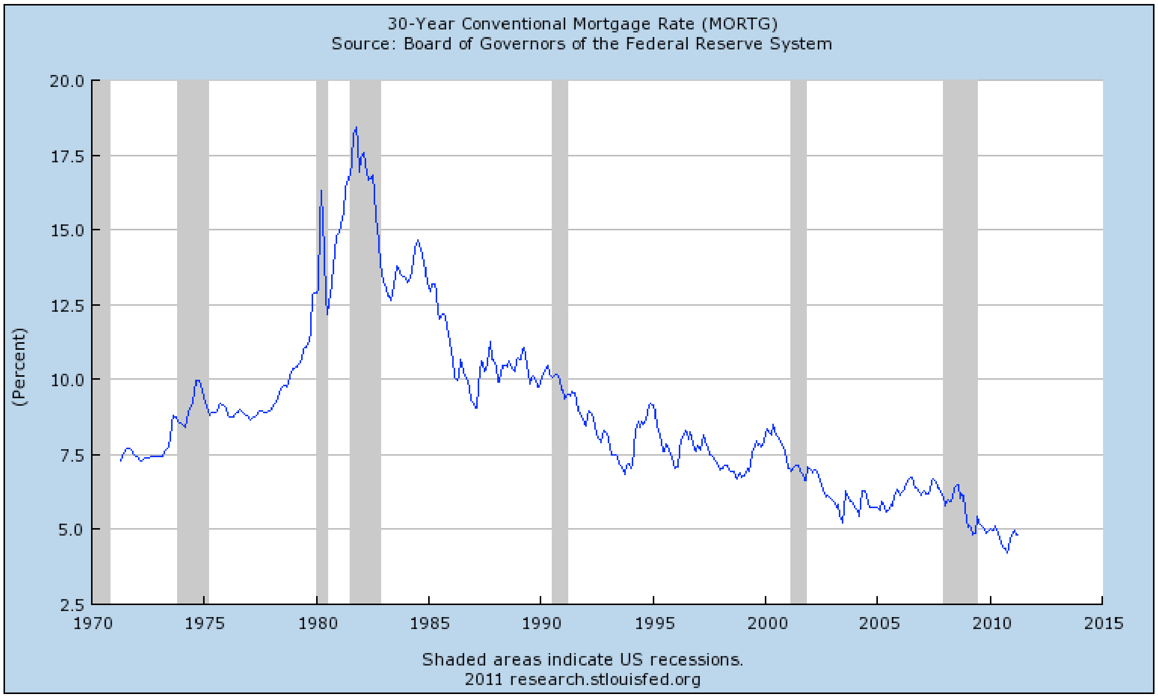

Figure 3.3. 30-year conventional mortgage rate

Chapter Three The Failure of the Quants

3.1. *Theme of this chapter*.

According to the Efficient Market Hypothesis (EMH), it is impossible to know when a bubble is occurring, since current prices reflect all publicly available information. Only in retrospect, when the bubble bursts is it clear that the prices were inflated. I show that the EMH did not describe accurately what happened. Quants missed systemic risk because they ignored publicly available information concerning the mortgage market. That market was very important in the highly leveraged and interrelated financial markets. The collapse of the mortgage market bubble led to the financial collapse.

Part 3.2 is a discussion of leveraging, and leads into a vivid example of the strategy of the Atlas Fund run by Quants, which was given AAA ratings. At first, this hedge fund was extremely profitable and then it collapsed. This example illustrates why the Quants failed. Part 3.3 describes the structure of the derivatives market, the securitization, interrelationships, the rating agencies and pricing of derivatives. Part 3.4 describes the crucial underlying models used by the Quants based upon the No Arbitrage Principle NAP - the CAPM, Black-Scholes-Merton (BSM) and EMH - in devising their instruments and strategy. Part 3.5, discusses the models, based upon the No Arbitrage Principle NAP, in detail. Part 3.6 concerns methods to determine when the drift of the capital gains has changed. The conclusion is part 3.7. It summarizes the errors made by the Quants in the pricing of the tranches and the Credit Default Swaps. The next few chapters explain why Stochastic Optimal Control (SOC) uses information much better and can provide Early Warning Signals (EWS) of a crash.

3.2. Leveraging

It is now widely believed that "'excessive" leveraging, and an "excessive" debt ratio, at key financial institutions helped convert the initial subprime turmoil in 2007 into a full blown financial crisis of 2008. Leverage is the ratio of assets /net worth $A(t)/X(t)$, or equivalently the ratio of debt $L(t)$ to net worth $X(t)$ denoted $f(t) = L(t)/X(t)$. Although leverage is a valuable financial tool, "excessive" leverage poses a significant risk to the financial system. For an institution that is highly leveraged, changes in asset values greatly magnify changes in net worth. To maintain the same debt ratio when asset values fall either the institution must raise more capital or it must liquidate assets.

The relations are seen through equations (i) – (iv). In (i) net worth $X(t)$ is equal to the value of assets $A(t)$ less debt $L(t)$. Equation (ii) is just a way of expressing the debt ratio. Equation (iii) relates the debt ratio $f(t) = L(t)/X(t)$ to the leverage ratio $A(t)/X(t)$ of assets/net worth. Equation (iv) states that the percent change in net worth $dX(t)/X(t)$ is equal to the leverage $(1+f(t))$ times $dA(t)/A(t)$ the percent change in the value of assets.

(i) $X(t) = A(t) – L(t)$.

(ii) $L(t)/X(t) = f(t) = 1/[(A(t)/L(t) – 1]$.

(iii) $A(t)/X(t) = 1 + f(t)$.

(iv) $dX(t)/X(t) = (1+ f(t))\, dA(t)/A(t)$.

The Congressional Oversight Panel (2009) COP reported that, on the basis of estimates just prior to the crisis, investment banks, securities firms, hedge funds, depository institutions, and the government sponsored mortgage enterprises – primarily Fanny Mae and Freddie Mac - held assets worth \$23 trillion on a base of \$1.9 trillion in net worth, yielding an overall average leverage of $A/X = 12$. The leverage ratio varied widely as seen below.

Table 3. 1. Leverage of Institutions

Broker-dealers and hedge funds	27
Government sponsored enterprises	17
Commercial banks	9.8
Savings Banks	6.9
Average, weighted	12

Consider the average leverage, where $A(t) = \$23$ trillion, $X(t) = \$1.9$ trillion, $L(t) = \$21.1$ trillion, making the debt ratio $f = 11.1$. From equation (iv), a 3% decline in asset values would reduce net worth by $dX(t)/X(t) = (1+11.1)(0.03) = 36\%$. The loss of net worth is equal to (0.36)($1.9 trillion) = $0.69 trillion. To maintain the same ratio $f = 11$, the institutions must either raise capital to offset the decline in asset values $dX = dA < 0$, or must sell off assets to reduce debt by the same proportion $dL(t)/L(t) = dX(t)/X(t)$, derived from equation (ii). A 3% decline in asset value would require the sale of (0.03)(21.1 trillion) = $630 billion in assets to repay the debt.

Both actions have adverse consequences for the economy. Firms in the financial sector, the financial intermediaries, are interrelated as debtors-creditors. Banks lend short term to hedge funds who invest in longer term assets and who may also buy credit default swaps. Firms that lost $690 billion in net worth would have difficulty in raising capital to restore their net worth, without drastic declines in share prices. Similarly, the attempt by one group to sell $630 billion in assets to repay loans will have serious repercussions in the financial markets. The prices of these assets will fall, and the leverage story repeats for other groups. Institutions who hold these assets will find that the value of their portfolios have declined, reducing their net worth. In some cases, there are triggers. When the net worth of a Fund falls below a certain amount ("breaks the buck") the fund must dissolve and sell its assets, which may include AAA assets. In turn the sale of AAA assets affects other institutions. Conservative investors may have thought they were holding very safe assets, but to their dismay they suffer capital losses as AAA assets are liquidated at "fire sale" prices. In a highly interrelated system, "high leverage" can be very dangerous. What seems like a small shock in one market can affect via leverage the whole financial sector. The Fed and the IMF seemed oblivious to this systemic risk phenomenon because of the history of two previous bubbles. In the S&L and agricultural crises of the 1980s, discussed in chapter seven, there was not a strong linkage between the specific sector and a highly leveraged interrelated financial sector based upon CDO and CDS. Therefore the collapse of these earlier bubbles only had localized effects.

3.2.1 The Incredible Leverage of Atlas Capital Funding

The story of the Atlas Capital Fund is an excellent example of leveraging discussed above. My discussion is based upon a paper given by Jichuan Yang, one of the principals of Atlas, given at an Applied Mathematics Colloquium at Brown University in

September 2009 and the paper by Ren Cheng (former Chief Investment Officer at Fidelity) at the same Colloquium. A group of talented financial engineers: mathematicians, physicists specializing in mathematical finance, decided to establish a Fund in 2003 with $12 *billion* of assets, and $10 *million* of capital, - a leverage of 1200. This Fund was called the Atlas Capital Fund, due to its huge size. The fund portfolio would contain thousands of individual bonds, loans and other financial securities that had longer term maturities, such as 8 years. The liabilities were commercial paper and mid-term notes with maturities ranging from 30 days to 5 years. Atlas would borrow short term and lend longer term to the Hedge Funds. The Fund was not set up to hedge risk but to seek maximum return. The Fund did not fear taking risk. Atlas would make its profits from the difference between the lending rate charged to the hedge funds and the cost of short term borrowing. The latter could be reduced to a minimum if Atlas received a AAA rating. This was a remarkable goal because most global banks are rated no higher than AA.

Since the portfolio had a much longer maturity than the loans, a major risk to Atlas would be the variable short term borrowing rate. Figure 3.2 graphs the volatility of the short term interest rate. When the 30-day loan matured, Atlas would roll over the 30-day loan at the current rate. If there were difficulties in rolling over, Atlas would have to find banks, called “liquidity providers”, to give Atlas “emergency” loans to pay off the 30-day debt.

The “financial engineers” built a model to evaluate the risk, which they used to convince the rating agencies to give them an AAA rating, which lowers the cost of borrowing. The model simulated the movement of the $12 billion of individual assets as well as their correlated behavior. The mismatch of the timing of cash flows of assets and liabilities, the price movements, the rating changes, the defaults and recovery had to be “accurately” modeled, calculated and simulated. For each potential future price movement, the model calculated the gain, loss and return. After tens of thousands of such simulations, the financial engineers arrived at an estimate of the expected loss and expected return by certain types of averaging the individual outcomes. These simulations convinced the rating agencies to give Atlas an AAA rating and hence a low cost of borrowing.

At first, Alas was extremely profitable. Stockholders received 100% of their money back in the first year of operations. due to the leverage of 1200*,* equal to $12 billion of assets/$10 million of capital. The Fed was most accommodating with its low interest policy. Moreover, Chairman Alan Greenspan was the champion of financial innovation and was fighting off regulatory reform on all fronts, as discussed in chapter two. About three years after Atlas started operations, the US financial industry went into one of its worst crises. The cascading effects of leverage discussed above then occurred. Atlas was blamed as being one of the main culprits causing the crisis. Jichuan Yang, a principal of Atlas, wrote in 2009: “Today, if someone tells me that all these things can be simulated by an elegant mathematical model with any realistic accuracy, I would be tempted to say that he’s probably an overconfident idiot”.

3.3. Structure of Derivatives Market, Rating Agencies and Pricing of Derivatives

This part draws upon the comprehensive FCIC report, and the books by Derman and by Patterson. The financial market consisted of several stages. At one end were the mortgagors, the households who borrowed against negligible collateral or ability to service the loans from income, the NINJAs (no income no jobs, no assets). Their loans were packaged or securitized into bundles by financial intermediaries who could not perform due diligence because they had no idea of the quality of the loans. In turn, these packages were sold to institutional investors, who relied upon the rating agencies to evaluate risk. This method was used by FNMA and then followed by the private sector.

Vast quantities of similar but not identical securities were pooled into bundles and sold to large investors, mutual funds, pension funds, insurance companies and hedge funds who were seeking high return and low risk investments. There was no clear way to evaluate the risk of a package, because it consisted of many mortgages of dubious quality. However, it was believed that by pooling the securities the risk would be diversified. There would be losses on the poor quality ones but the returns on the high quality mortgages would be uncorrelated with the poorly performing group. That is, the bundle would be like “*apples in a basket*”. There will be some rotten ones, but there would also be good ones. The process of asset acquisition, pooling and standardization would be attractive to investors. There was a belief in geographic diversification and that real estate markets are local ones/independent. This belief reflected the fact that nationwide real estate price declines had not been experienced since the Great Depression. The banks, the original owners of the mortgages, sold them to the investment houses that securitized the packages, and could make more loans. The supply of mortgages would then be increased, just as the demand was increased by the risk diversification packages. Greenspan believed that: “By far the most significant event in finance during the past decade has been the extraordinary development and expansion of financial derivatives.”

The Quants devised a method to sell the derivatives of dubious value as AAA securities. A prioritization method was used. Tranches of the pool were sold, like bonds, preferred stock and common stock *in the same bundle*. Losses or defaults would first affect the lowest tranche (Equity) then when that tranche’s assets were exhausted, the losses would affect the medium (Mezzanine) tranche. The top, senior tranche (AAA) would only lose if the losses exceeded the assets in all the tranches.

Ratings were not based upon the quality of the underlying mortgages all of which were in the same bundle. The rating depended upon who took losses first in the stack of loans. The rating agencies were essential to the smooth functioning of the mortgage backed securities market. Banks needed their ratings to determine the amount of capital to hold, repo markets needed the ratings to determine loan terms, and some investors could only buy securities with a AAA rating. Credit ratings also determined whether investors could buy certain investments. The SEC restricts money market funds to purchasing securities that have received credit ratings in one of the two highest short-term rating categories. The Secondary Mortgage Market Enhancement Act of 1984 permitted federal and state chartered financial institutions to invest in mortgage related securities if the securities had high credit ratings from at least one rating agency. Many investors, such as pension funds and university endowments relied on credit rating agencies because they had neither access to the same data as the rating agencies nor analytical ability to assess the securities that they were purchasing.

The logic of trenching was the *“apples in a basket model”*. The “*loaf of bread”* model, discussed in my criticism below, was not considered. Say that there were two tranches A(AAA, top) and B (Equity,bottom). Let $1 > p > 0$ be the probability of default of the entire package. The losses first affect tranche B. Tranche A can only suffer losses if tranche B is wiped out.

The probability Pr(B) of default of tranche B is $1 > p > 0$. The probability that that both tranches suffer losses is Pr(AB). In the *Apples in a Basket* case the defaults in the two tranches are *independent samples* of apples from a population (basket) where the probability of default is p. A first apple (Equity or Mezzanine tranche) is taken from the basket and it turns out to be rotten. This will occur with probability p. A second apple (Senior tranche) is taken out of the basket. In the "basket of apples" case, the $Pr(A|B) = Pr(B) = p$. In this case of independence, the probability that both apples are rotten corresponds to the case where the senior tranche would be exhausted $Pr(AB) = Pr(A)Pr(B) = p^2 < p$. Risk of the senior tranche is less than that of the entire package. The senior tranche appeared to be a silk purse made from a sow's ear.

Since the mid 1990s Moody's has rated tranches of mortgage backed securities using several models. Although Moody's did not sample or review individual loans, the company used loan/value ratios, borrower credit scores and loan terms. The model simulated the performance in 1250 scenarios including variations in interest rates and state unemployment rates as well as in home price changes. Thereby ratings were given to the tranches.

On average across the scenarios, home prices trended upwards at approximately 4% per annum. The model put little weight on the possibility that prices would fall sharply nationwide. Even as house prices rose to unprecedented levels, Moody's never adjusted the scenarios to put greater weight on the possibility of a decline. This choice did not consider that there was a national housing bubble and did not sufficiently account for the deterioration of the quality of loans securitized. In October 2007, Moody's downgraded all of the mortgage backed securities that it had rated AAA in 2006, and downgraded 73% to junk. There was indeed a house price bubble. The obvious question is why was the rating system a failure?

3.3.1 Pricing CDOs

Ratings of the tranches were not based upon the quality of the underlying mortgages. They were all in the same bundle. The rating depended upon who got paid first in the stack of loans. The key question was how to rate and price the tranches. The issue concerned the correlation of the tranches. If a pool of loans started experiencing difficulties, and a certain percent of them defaulted, what would be the impact upon each tranche? The "apples in the basket model" made one prediction. Another very different one is "the slice of bread in the loaf" model.

The *"slices of bread"* model assumes systemic risk, in which the value/returns of assets are very highly or perfectly correlated. The probability of having moldy slices in the package is $1 > p > 0$. If the first slice (B) taken from the loaf is moldy, what is the probability that the second one (A) is also moldy? The *conditional* probability that assets in tranche A (second adjacent slice is moldy) default, given that B defaults (first slice is moldy) is close to unity, $Pr(A|B) = 1$. In this case, the probability that the owners of tranche A lose is $Pr(AB) = Pr(A|B)\,Pr(B) = 1.Pr(B) = p$. The senior tranche is as risky as is the average of assets in the portfolio. Tranching/securitization can allow risk reduction only if there is no systemic risk. With systemic risk, where $Pr(A|B) = 1$ or close to it, the whole structure can collapse. If B defaults one is confident that A will default.

Since there were many assets in the CDO and no due diligence was performed, it was extremely difficult or impossible to evaluate the correlations or conditional probabilities of defaults among tranches. Because there was no organized exchange for the CDOs, their market prices were not transparent. No one knew what they were worth if sold on the market. This was the challenge for the Quants and rating agencies.

The "solution" was Li's copula/CDS model. One would know if they viewed "the apples in the basket" or "slices of bread" case as more relevant. In the "slices of bread" case, there was no point in correlating tranches. Only the probability p that the package will default is relevant.

The CDS rate was linked closely with house prices. Because the CDO boom was occurring at the same time as the housing prices were inflating, the CDS showed very little risk. In fact, there was a positive feedback between the CDS rate and housing prices.

The rise in house prices lowered the CDS rate on the tranches and the packages of tranches increased in value. This increased the demand for mortgages by the securitizers, which in turn induced a greater supply of mortgages of any quality. Thereby housing prices rose further.

3.4. Major Premise of Economics/Finance: No Arbitrage Principle (NAP)

The Quants relied upon three models: the CAPM, the Black-Scholes-Merton Options pricing model (BSM) and the Efficient Market Hypothesis (EMH) to price securities and manage risk. The first two models are logical deductions from their premises. The BSM was a brilliant synthesis of mathematics and economics. Both the CAPM and BSM were based upon the major premise in economics and finance: the "*No Arbitrage Principle*" (NAP).

The NAP is the foundation of microeconomic household and firm optimization. A main principle in economics is that the gain per dollar spent on a consumer good should be the same for all goods consumed. Similarly in the theory of the firm, the gain per dollar spent on an input in the production process should be the same for all inputs used. The NAP conditions are expressed as follows. Let the consumer have a utility function over n-goods or services. The price of each good is known and fixed. The budget constraint is that the sum of expenditures over the n-goods equals the given budget. The NAP equation is that the marginal utility per dollar should be the same for each of the n-goods consumed. The marginal rate of substitution in utility (along the indifference curve or surface) should equal the relative given market prices (slope of budget line). Utility is thereby maximized subject to the budget constraint. If the relative marginal utilities are given, the relative prices are determined, and vice-versa.

In the theory of the firm, a known production function relates inputs to output. For a given cost, the inputs used should be such that their marginal product per dollar of input price be equal for each input used. Equivalently, the marginal cost to produce output (price input /marginal product) should be the same for all inputs used. If one knows the relative marginal productivities, then relative input prices are known and vice-versa. This no arbitrage condition NAP minimizes costs for a given output.

3.4.1. CAPM Model

The no arbitrage condition takes an analogous form in CAPM and Black-Scholes-Merton models in finance. The NAP principle in the CAPM was derived by Sharpe (1964) as follows. Let there be a market (m) portfolio of n assets. Each asset has a return and a risk/variance, and there are n^2 variances and covariances. The market portfolio (designated by "m") has an expectation μ and a variance σ_m^2, based upon the variance/covariance matrix of the n-assets. There is also a safe asset with an interest rate of r. One can achieve a combination of expected returns and risk by combining the market portfolio whose return- risk is (μ,σ_m) with the safe asset whose return- risk is (r,0). This linear combination is the capital market line, where the risk-return trade-off is:

$$dE/d\sigma_m = (\mu - r)/\sigma_m. \tag{3.1}$$

One can also achieve a risk- return by varying the composition of the market basket. Let there be a *portfolio* where fraction a is in asset i and fraction (1-a) is in the entire market basket of all n = 1,2,…i,…n-assets. The expected return is E and the variance is σ^2 for the portfolio, equation (3.2).

$$E = aE_i + (1-a)\mu \qquad variance = \sigma^2 \tag{3.2}$$

As fraction *a* varies, both the expected return and the risk of the portfolio change. One obtains dE/da and dσ/da by varying the composition. Eqn. (3.3) is the risk-return trade-off obtained by varying the composition of the portfolio, *evaluated at the optimum portfolio*, when a = 0.

$$\{[(dE/da)/ (d\sigma/da)]|a=0\} \tag{3.3}$$

The NAP requires that the two risk-return trade-offs be equal. Eqn. (3.4) equates eqn. (3.1) to eqn. (3.3).

$$\{[dE/d\sigma_m] = (\mu - r)/ \sigma_m \} = \{(dE/da)/ (d\sigma/da)]|a=0\} \tag{3.4}$$

From (4) one obtains the CAPM equation (3.5).

$$[E_i - r] = \beta_i[\mu - r], \qquad \beta_i = \sigma_{im}/\sigma^2_m \tag{3.5}$$

The CAPM provides a good measure of risk. Assets can only earn high average returns if they have high betas. Average returns are driven by beta because beta measures the extent that the addition of a small quantity of the asset to a diversified portfolio adds to the volatility of the portfolio. Beta is estimated by regressing the observed return of asset i upon the observed return on the total portfolio, each adjusted for r, the safe rate of return.

3.4.2. BSM Model

The BSM options pricing is based upon the NAP. Fischer Black derived the equation on the principle: Assets must be priced such that risk per unit of return is the same for all traded assets. Let there be a stock whose price S(t). The price change dS(t) is the sum of a drift plus a Brownian Motion (BM) term dw(t), where E(dw(t)) = 0, $E(dw(t))^2 = dt$. The drift coefficient μ and diffusion coefficient σ are constant.

$$dS(t) = \mu S(t)dt + S(t)\sigma\, dw(t) = \text{drift } S(t) + \text{diffusion } S(t). \tag{3.6}$$

There is a derivative whose value V is linked to the price of the stock S(t). There are several ways to derive and view the BSM equation and I follow the Fleming-Soner (2006, 360-62) exposition within the NAP. One can buy the derivative at time t whose value V(t,S) depends upon the stock price at time t, or one can purchase for the same amount a portfolio of value X(t) = V(t,S), consisting of the stock and a bond yielding a safe return of r.

The change in the value of the portfolio dX(t), equation (3.7) is the sum of a drift term and a diffusion term. Similarly the change in the value of the derivative dV(t) is the sum of a drift and a diffusion term, equation (3.8).

$$dX(t) = \text{drift } X(t) + \text{diffusion } X(t) \tag{3.7}$$

$$dV(t, S(t)) = \text{drift } V(t) + \text{diffusion } V(t) \tag{3.8}$$

The NAP states that, for the same risk, the change in the value of the portfolio dX should equal the change dV in the value of the derivative dX(t) = dV(t, S(t)). Use equations (3.7) and (3.8) to derive NAP equation (3.9).

$$\text{drift } X(t) + \text{diffusion } X(t) = \text{drift } V(t) + \text{diffusion } V(t) \tag{3.9}$$

The NAP requires that both assets having the same risk must have the same return. Equate drift X(t) to drift V(t), eqn. (3.10). This equalizes the return. Equate the two diffusion terms, eqn (3.11). This equalizes the risk. The portfolio composition at each time must be adjusted to provide an equal risk.

$$\text{drift } X = \text{drift } V \tag{3.10}$$

$$\text{diffusion } X = \text{diffusion } V \tag{3.11}$$

Solve the two equations for the ratio of the stock/portfolio and for the value of the derivative V. The BSM equation (3.12) for derivative pricing V(t,S(t)) is derived.

$$V_t(t,S) + rSV_s(t,S) + (1/2)\,\sigma^2 S^2 V_{ss}(t,S) = rV(t,S). \qquad (3.12)$$

The main implications of BS are: (a) the drift term μ of the stock plays no role in the pricing. (b) The variance σ^2 plays a very important role.

3.4.3. The Efficient Market Hypothesis (EMH)

An important hypothesis used in the finance literature is the EMH Efficient Market Hypothesis. It is based upon the hypothesis that "Properly Anticipated Prices Fluctuate Randomly". However, its application to the world of finance with the derivatives rests upon vague and arbitrary assumptions. This was the fatal flaw. First I discuss these models and then evaluate their application to the pricing of derivatives.

The EMH is an empirical application of the hypothesis that "Properly Anticipated Prices Fluctuate Randomly" [Samuelson (1965)]. The latter is a set of mathematical propositions. The EMH has been applied to stock prices and foreign exchange rates with less than successful verification. I explain later in this chapter why the attempt to apply it pricing of CDOs, CDS and tranches failed and led to the bubble and its collapse.

The view underlying the EMH is that in an "informationally efficient" market price changes must be unforcastable. By "informationally efficient" one means that the market price fully incorporates the expectations and information of the market participants [Fama (1970)]. The mathematical structure of the argument below is based upon Feller, volume II (1966). It consists of three propositions, any one will imply the other two.

Let X(t) be the change in price P(t). Equation (3.13) states that the expectation of the change in price E[Xt)] = 0. This proposition is called an absolutely fair game.

$$E[X(t)] = 0. \qquad (3.13)$$

It reflects the view that, in a competitive market with informed buyers and sellers trading at market determined prices, if one were sure that a price will rise, it will already have risen.

Price P(t) is the sum of previous price changes X(s), s < t. It can be expressed as the previous price P(t-1) plus the current price change X(t), equation (3.14).

$$P(t) = \Sigma(X(s)) = P(t-1) + X(t). \qquad t \geq s \geq 0. \tag{3.14}$$

The expectation of the price E[P(t)|P(t-1), P(t-2)…], conditional upon its past history,

$$E\,P(t) = E\,[\,\Sigma(X(s)] = P(t-1) + E\,X(t) = P(t-1) \tag{3.15}$$

is equation (3.15), because equation (3.13) states that the expectation of a price change E X(t) is zero. This proposition is the *martingale property*. The expectation of the next period's price is the current price.

The third proposition is called the *Impossibility of Systems*. It states that no strategy based upon the past history of prices can be profitable. No system can produce positive expected profits. Although a system may work at one time, that is a fluke and what works once is unlikely to work again.

Z(t) is cumulative profits. Let v(t) be a system, a formula, fixed in advance. It tells one when and how much to buy/sell at any time, based upon the past history of prices or any other variables. Then Z(t)

$$Z(t) = Z(t-1) + v(t)[P(t) - P(t-1)] = Z(t-1) + v(t)X(t). \tag{3.16}$$

is the sum of the previous cumulative profits Z(t-1) plus the profit from the last trade, based upon the fixed in advance system v(t).

The expectation of cumulative profits E[Z(t)] conditional upon the past prices is

$$E[Z(t)] = Z(t-1) \qquad \text{since } E[X(t)] = 0 \tag{3.17}$$

It is equal to the previous profits, since the expected price change EX(t) = 0 from equation (3.13). Any sequence of decision functions converts martingale {P(t)} into martingale {Z(t)}.

Any one equation (3.13),(3.15), (3.17), which describes the hypothesis that "Properly Anticipated Prices Fluctuate Randomly", implies the other two. This is a mathematical proposition but it does not explain where the basic probability distributions underlying Expectation *E* come from. It does not explain what is "Information" nor does it explain how market participants act upon what they think is "information" or by "risk adjusting" their expectations.

The next section discusses how the market/Quants used these hypotheses in the world of CDOs, CDS, tranches and options, and how their misuse led to and aggravated the bubble.

3.5. The Quants and the Models

On the basis of the EMH and CAPM, Greenspan, the Fed and the finance profession believed that markets would be self-regulating through the activities of analysts and investors. In their view, government intervention weakens the more effective private regulation.

Securitization/tranching, the various layers of CDOs, and CDS, produced an environment where the EMH/CAPM lost relevance. These bundles of many mortgage based securities seemed to tailor risk for different investors. Securitization/tranching gave the illusion that one could practically eliminate risk from risky assets and led to very high leverage, as discussed in the section on the Atlas fund. In hindsight, it was difficult to understand how tranching the equity tranche- which consisted of poor prospects - into tranches could produce AAA ratings.

In reference to the LTCM collapse, Derman (p. 190) wrote: "It was a shock to realize that people whose great experience and knowledge straddled both the quantitative and trading worlds had, despite their sophistication, brought themselves into such a catastrophic state". The same could certainly be said about the financial collapse of 2007-2008. How can the latter be explained?

The Quants made several serious errors in modeling. First, there were no relevant "betas" to measure the risk of the CDOs. Second, the martingale property of the price is a special case of "The Principle that Properly Anticipated Prices Flucutate Randomly", which does not apply to the house price index. Third, they ignored the "no free lunch" principle, discussed below, that the expected present value of an asset must be finite. Fourth, they assumed that there is a stable distribution function for house prices, from which they could derive the VaR value at risk. They assumed that the "free lunch" would continue, and invoked a "Black Swan" to justify their failures Fifth, they ignored the feedback in both directions between house prices, the price of a CDS, and the debt of households. Each of these is discussed in turn.

3.6.1. The CAPM

The financial market consisted of several stages. At one end were the mortgagors, households who borrowed against negligible collateral or ability to service the loans from income. Their loans were packaged or securitized into bundles by financial intermediaries who could not perform due diligence since they had no idea of the quality of the loans. In turn these packages were tranched and sold to institutional investors, who relied upon the rating agencies to evaluate risk. How could the risk be evaluated? The CAPM states that it is measured by the beta of the tranche, eqn. (3.5) repeated.

$$[E_j - r] = \beta_j[\mu - r], \qquad \beta_j = \sigma_{jm}/\sigma^2_m$$

To calculate the beta of tranche j one must know, the expected return μ on the larger portfolio containing tranche j, the variance-covariance matrix of all of the tranches and assets in the larger portfolio, the covariance σ_{jm} to between the return on tranche j and the return on the larger portfolio. These quantities could not be calculated. There were thousands of mortgages in each tranche, most of them of dubious value. No due diligence was performed. Their market prices were unknown so that one could not calculate the E_j the expected return on the tranche. In addition, what was the market portfolio containing tranche j whose mean was μ, whose variance was σ^2_m and covariance was σ_{jm} with tranche j?

3.5.2. Credit Default Swaps, EMH and the House Price Index.

Since there are many assets in the CDO and no due diligence was performed, it was extremely difficult or impossible to evaluate the correlations or conditional probabilities of defaults among tranches. There was no organized exchange for the CDOs. Their market prices were not transparent. No one knew what they were worth if sold on the market and internal accounting rules allowed Mark-to-Market values to be set by internal risk models rather then by verifiable quotes from transaction. This was a challenge for the Quants and rating agencies.

The “solution” was Li’s copula/CDS model. His model could assign correlations between the tranches by measuring the Credit Default Swap rate (CDS) linked to the underlying security. The CDS of the tranches supplied a single variable that incorporated the market’s assessment of how the tranche will perform. The CDS rate was linked closely with the prices of the mortgages and CDOs, which in turn was based upon the house price index. Then one could price the CDOs/tranches based upon the CDS rate. The important question was how to estimate the expected house price, in order to price the CDS.

EMH equation (3.15) states that the expected price EP(t+dt) is equal to the present price P(t), the martingale property. This is only true, however, in special cases and is not implied by the NAP. For example, the spot price of a harvest commodity is not a martingale, whereas the futures price is a martingale, [Stein (1986), Samuelson (1957)].

Inventories rise as the harvest comes in. The balance of short hedging by those carrying inventories lowers the spot price below the price expected at a later date, due to a risk premium and storage costs. As inventories decline, short hedging declines and the spot price rises towards the expected price. Thus P(t) is less than EP(t+dt) and its expected change E[dP(t)] is not zero.

Similarly, before the harvest there is a balance of long hedging, by those who want to protect themselves from later purchases at higher prices . The spot price P(t) rises above the expected price, due to a risk premium and very low inventories. As the harvest comes in, there is a decline in long hedging, and the spot price declines. Thereby P(t) is greater than EP(t+dt) and its expected change E[dP(t)] is not zero. The NAP is not violated. In both cases however, the futures price is tied to EP(t+dt) and is a martingale.

In the case of the CDS, the variable to consider is the index of house price P(t). Let the house price follow a random walk with drift, equation

$$dP(t) = \mu P(t)dt + P(t)\sigma\, dw(t) = drift + diffusion,\ E[dw] = 0,\ E[dw^2] = dt. \qquad (3.18)$$

The drift coefficient, the trend, is μ and the diffusion is $\sigma\, dw(t)$. The CDS rate is based upon the expected house price index. Solve (3.18) for P(t), using the Ito equation to derive (3.19). Take the expectation and derive equation (3.20).

$$P(t) = P(0)\ exp\{ \int [(\mu - (1/2)\sigma^2]dt + \int \sigma\, dw\}\ \ t > 0. \qquad (3.19)$$

$$EP(t+dt) = P(t)\ exp\{ \int \mu\ \ d\tau\},\ t+dt > \tau > t. \qquad (3.20)$$

The expected house price that determines the CDS rate is equation (3.20). *This is a rational expectations price.* Only if μ = 0, there is no drift in the house price index, would the house price and the underlying CDS be a martingale.

The CDS rate was then used as a measure of risk of the tranche. One must have estimates of μ to form an estimate of the expected house price that determines the CDS rate and price/risk of the tranche. Since the CDO boom was occurring at the same time as the housing prices were inflating, the CDS were showing very little risk according to this view.

To estimate the trend μ or drift the Quants would have to estimate the distribution of house prices. Then they could use a VaR to determine the probability of a decline in house prices. There were several possibilities. (i) Assume a stable distribution of house prices, based upon historical data; (ii) Assume an unstable, a changing, distribution ; (iii) Assume a stable Pareto-Lévy distribution which has a fat tail; (iv) Assume a jump diffusion process.

Figure 3.4 is a histogram of the distribution of [P(t) – P(t-1)]/P(t-1) the percentage change in house prices, the capital gains – over the period 1980q1 – 2007q4 prior to the crisis. These data could be used to infer the drift and risk parameters underlying the expected price. A VaR could then be derived.

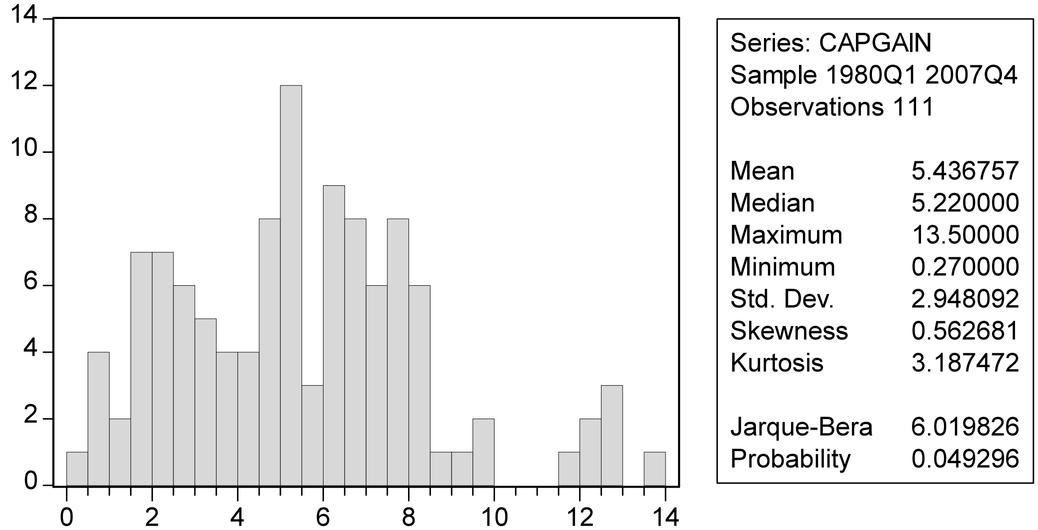

Figure 3.4. Histogram and statistics of CAPGAINS = Housing Price Appreciation HPA, the change from previous 4-quarter appreciation of US housing prices, percent/year, on horizontal axis. Frequency is on the vertical axis. Period 1980q1 – 2007 q4. Source: Office of Federal Housing Price Oversight.

The histogram shows several factors. (1) Estimates of trend μ = 5.4% pa and of risk σ = 2.9 % pa. (2) The null hypothesis is that distribution is normal. The probability is about 5% that the Jarque-Bera statistic exceeds the observed value under the null hypothesis. Small probability value leads to the rejection of the null/normality. (3) There were no sub-periods of falling house prices. (4) There is a positive tail. These were the "bubble years".

Figure 3.5 is a time series of capital gains [P(t) – P(t-1)]/P(t-1) CAPGAIN and the debt/disposable income of households DEBTRATIO, over the period 1980q1 – 2007q4. A major error that the Quants made was to assume that the distribution described by the histogram figure 3.4 is stable. They assumed that these capital gains could continue while the debt/income ratio of households was rising steadily relative to its longer run mean value. The distribution of the capital gains was very different in the period 2000 – 2011.

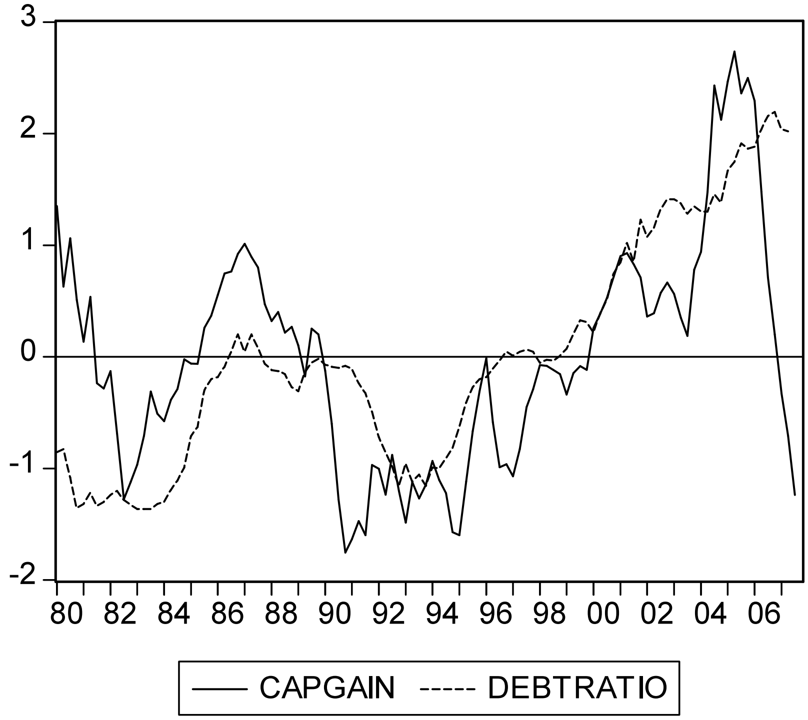

Figure 3.5. Capital gains [P(t) – P(t-1)]/P(t-1) = CAPGAIN; DEBTRATIO = household debt/disposable income. Variables are normalized to a mean = 0 and a standard deviation = 1.

The capital gains from 2004-2006 reached 2.9 standard deviations above the mean for the entire period 1980q1 to 2007q4. The mortgage interest rates were between 5 and 7 percent per annum as seen in figure 3.3. Thus during the 2004-06 period the capital gains exceeded the mean interest rate. If an estimate of the drift term μ or expected capital gain exceeds the mean interest rate r, then the expected present value PV of the asset $PV(t) = P(0) \exp [(\mu - r)t]$ diverges as time increases. That makes no sense.

Similarly, the Quants ignored the fact that many mortgagors were enjoying a "free lunch". The mortgagor had a mortgage of M(1), say equal to the value of the house, at interest rate r(t) for one "period". At the end of the period, the debt was M(1)(1+r(t)). Insofar as housing prices rose by $\pi(t)$ percent over the period, the value of the house was $M(2) = M(1)(1+\pi(t))$. Insofar as the capital gain $\pi(t) > r(t)$, the mortgagor refinanced and took out a mortgage equal to M(2), against equity. He repaid the loan and used the proceeds $M(1)(\pi(t) - r(t))$ for expenditure. This procedure was continued/repeated as long as the capital gain $\pi(t)$ exceeded i(t) the interest rate. The difference $M(t)(\pi(t) - r(t))$ was a free lunch that could not continue. As soon as the capital gain fell below the interest rate, the value of the house would be less than the debt. Delinquency and foreclosure would follow. This indeed ccurred.

3.6. When has the drift changed?

When the collapse occurred some Quants tried to adjust the stochastic process by assuming a jump diffusion process. To a constant diffusion of prices they assumed that there was a small probability that the price might take a sizeable jump. This ad hoc procedure is not useful. How can one estimate this small probability, especially since there were very few observations of a crisis? Figure 3.4 shows that there were no price declines from 1980 until the 2007 crisis. From the time series and distribution of prices, there is no information of when or by how much the jump will occur. There is no early warning signal of a jump or when the drift has changed.

A much more rigorous and elegant approach to the problem "When has the drift changed?" is the article by Blanchet-Scalliet et al. (Bl-S2007. An investor divides his wealth between a safe asset and a stock – the risky asset. The price of the stock has a drift and a diffusion. Their paper derives an optimal stopping or switching rule, given that the drift will change at some unknown time. The problem is to find an "alarm signal" for when to take some action.

To relate their work to this chapter let the price of the risky asset be the house price index P(t). The risky asset is a CDO or CDS whose price is linked to the house price P(t). The house price P_t evolves according to Eq. (3.21) – (3.25), where μ_t is the drift and $\sigma\, dw$ is the diffusion. The past/current drift is μ_1 and we anticipate that it will change to μ_2 at an unknown later date from the present. The expected drift is $E\mu_t$.

$$dP_t = P_t(\mu_t\, dt + \sigma\, dw) \qquad P = P(0) \qquad dw \sim N(0, dt) \qquad (3.21)$$

$$d \ln P_t = (\mu_t - \sigma^2/2)dt + \sigma\, dw_t \qquad (3.22)$$

$$\ln P_t = \ln P + \int(\mu_s - (1/2)\sigma^2)\, ds + \sigma\, w(t) \qquad w(t) = \int dw_s\, ds \qquad (3.23)$$

$$E \ln P_t = \ln P + \int (E\mu_s - (1/2)\sigma^2)\, ds \qquad (3.24)$$

$$var \ln P_t = \sigma^2\, t \qquad (3.25)$$

Bl-S postulate the *unconditional* probability, that there is no change in the drift, is Eq. (3.26a), and the *unconditional* probability of a change is (26b). If Pr $(\mu_t = \mu_1) = 0.5$, then the half-life in state μ_1 is eqn (3.26c). Thus $\lambda > 0$ is inversely related to the half- life in state 1.

$$Pr\,(\mu_t = e^{-\lambda t}) \qquad t \geq 0 \qquad (3.26a)$$

$$Pr\,(\mu_t = \mu_2) = 1 - e^{-\lambda t}. \qquad (3.26b)$$

$$\ln(0.5)/\lambda = 0.69/\lambda = \text{half-life} \qquad (3.26c)$$

From (3.21-3.26) how should one determine if drift μ has changed? This is an important question since it determines the optimal ratio of risky assets/wealth and the implied optimum growth rate of wealth.

They contrast three approaches. In one, the optimization is based upon a misspecified model, for example where future changes are ignored. The second is empirical. A moving average of prices is used to estimate when the drift has changed. Chartist trading rules are just based upon the history of the asset price. The third are two model-detect strategies.

Their model dependent optimization procedure is directly related to the question when has the drift changed? This challenge is illustrated by figure 3.5 above concerning the capital gain. I sketch the Bl-S approach.

Utility function of wealth ln X(t), implies considerable risk aversion. The aim is to select the proportion π_t in the risky asset to maximize the expected logarithm of wealth ln X(T) to time T > 0. That is, π_t maximizes Eq. (3.27).

$$G(X) = \max E \ln X_T/X, \quad X = X(0). \tag{3.27}$$

X = wealth = value of risky asset + value of safe asset

Their constraint is that $1 > \pi > 0$. Assume that $\mu_1 > r > \mu_2$. The drift starts above the safe rate and then will fall below it at some unknown time. The question is when one should reduce or even get out of the risky asset. When will the bubble burst? The expected drift times the fraction of wealth in the stock is $\pi_t E\mu_t$.

Using the stochastic calculus Ito equation they derive Eq. (3.28) from (3.27),(3.22) and an equation for wealth, a linear combination of the stock and the safe asset.

$$E \ln X_T/X = \int^T [\, \pi_t E\mu_t - \pi_t^2\sigma^2/2]\, dt + rT \tag{3.28}$$

Hence the optimal proportion π^*_t to be held in the risky asset is Eq. (3.29).

$$\pi^*_t = \operatorname{argmax} [\, \pi_t E\mu_t - \pi_t^2\sigma^2/2] \tag{3.29}$$

This ratio maximizes the expected utility, the expected logarithm of wealth at time T.

Their goal is to find a stopping rule that detects at what instant τ the drift of the house price index has changed. Since the investor observes the house price P(t), but cannot observe the drift $\mu(t)$, this is a partially observed optimal stopping problem. A technique using nonlinear filtering formulas to reduce this partially observed problem to a completely observed problem was first used by Shiryayev and has now become standard. Bl-S et al compute the wealth of the trader who uses one of the two Model and Detect Strategies, relative to the Moving Average atheoretical approach. They ask. *"Is it better to invest according to mathematical Model and Detect strategy based upon a mis-specified model, or according to a strategy which is model free?"*

The main mathematical tool used to obtain these two stopping rules is the process F_t, the *conditional* probability that the (unknown) change point appeared before the running time t. For each procedure, the trader decides to invest his wealth in the risky asset when F_t differs from a given quantity defined in their paper.

The Bl-S equation (3.30) is the *conditional expectation* of the drift μ^*_t. Variable F_t, is the conditional probability that the change in the mean rate of return from has occurred by time t, given the observed house prices up to that time. If F = 1 then the jump down from

μ_1 to μ_2 has occurred. If F = 0, the jump has not yet occurred.. The mean rate of return μ_t is a hidden state with two possibilities, μ_1 or μ_2., Only one change in the state is allowed at an random exponentially distributed time τ. This is different from an *unconditional* expectation based upon (3.26a)(3.26b). Using (3.30)

$$\mu^*_t = [\mu_1 - \sigma^2/2) + (\mu_2 - \mu_1)F_t] \qquad (3.30)$$

in (3.29), they determine the optimal ratio of the risky asset/wealth π^*_t which maximizes expected logarithm utility given the stochastic process on the drift.

The evolution of F_t, the conditional probability that change has already occurred, is based upon a likelihood ratio and an *innovation* process. It depends upon both the observed house prices and the conditional probabilities F_s, for $s \leq t$. The details are in their article. For my purposes, the derived conditional probability has the following characteristics [Bl-S, eqs. A6, A7].

Start with a drift μ_1. One has a prior and a recent sample. The forecasting error compares the current price realization ln P(t) with what is expected from the prior drift $[\mu_1 - \sigma^2/2]t$. The difference is used to obtain the probability F_t that the change in drift from μ_1 to μ_2 has already occurred.

They conclude (p. 1366): "We can now address our main question: Is it better to invest according to a mathematical Model and Detect strategy based upon a mis-specified model or according to a strategy which is model-free? Due to the analytical complexity of all the explicit formulae that we have obtained for the various expected utilities of terminal wealth, we have not as yet succeeded to find a mathematical answer to this question… We therefore present numerical results obtained from Monte Carlo simulations to illustrate our comparisons."

Bl-S simulate the models and conclude (page 1367) " Our numerical study suggests that there is no universal solution to the problem of parameter mis-specification. It seems that when the drifts are high in absolute terms and in particular, when the upward drift is high, the performance of the Model and Detect strategies can be quite robust and superior to the one of the chartist trading strategy. However, their performance deteriorates rapidly when λ is strongly mis-specified and/or when the upward drift is not very high. Since the second drift is in fact the hardest to estimate due to the fact that we lack a priori information, we recommend caution before asserting that the Model and Detect strategies are superior to the technical trading rule. Indeed the Model and Detect strategies only offer a clear comparative advantage over the chartist trading rule in the presence of strong expected future trends".

3.7. Conclusion: Errors of the Quants

According to he EMH, it is impossible to know when a bubble is occurring because current prices reflect all publicly available information. Only in retrospect, when the bubble has collapsed, can one infer that prices did not reflect “the fundamentals”. This was precisely Greenspan’s view (chapter 2): the crisis was unpredictable. The theme of this book, in chapters below, is that Greenspan was in error.

The Quants/market made three interrelated errors. First, they based the expected house price upon the current price and drift according to eqn. (3.20). When the drift $\mu = 0$, the expected price was just the current price (EMH). A second closely related error is that they assumed that the distribution of capital gains is constant, so they could derive a VaR on that basis. The mistake was that the Quants ignored publicly available information in forming expectations, as discussed in chapter two. The ultimate determinant of the values of CDO’s was the ability of the mortgagors to service their debts. The income to the owners of the CDO’s and their values comes from the flow of income through the tranches starting with the mortgagors. It was public information that the mortgages were of most dubious value, no due diligence was performed in the ratings and the debt/income of the mortgagors - DEBTRATIO in figure 3.5 was rising. Hence it was improbable that the distribution of house prices would remain constant. This systemic risk was ignored by the Quants who just focused upon current prices.

A third major error of the Quants was to assume the independence of the returns to the tranches, the “apples in the basket” model, instead of the correlations in the “slices of bread in a loaf” model.

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