Math Modeling Projects!

- 1. Michaelis-Menton Equation

  As you may know, the Michaelis-Menton Equation (MME) is useful in biological kinetics. As a parmacokinetic (rate of processing drugs) law, it provides the basis for the equations modeling, for example, blood alcohol levels. In pharmacokinetics, the MME governs the rate at which the body processes a drug. The form of the equation is

  $$dx/dt = \frac{-Kx}{A + x}$$

  where x is the concentration of a drug in the body at time t, and K and A are positive constants. This differential equation is frequently approximated by simpler differential equations.

  For many drugs (cocaine for example), A is much, much larger that $x(t)$ at all times. A typical case might be that $A = 6$ and $x(0) = 0.0025$. Using the idea that, if $A >> x(t)$ then $A + x(t) \cong A$, rewrite the MME and solve it symbolically for your favorite value of $x(0)$.

  For other drugs (alcohol for example) $x(t) >> A$, e.g. $A = 0.005$ and $x(0) = 0.025$. What would $A + x(t)$ approximately equal now?

  If you assume that $K = 1$ and the values for A and $x(t)$ as above, graph the solution curves for these two approximations.

- 2. Reservoir and Droughts

  Reservoir are lakes that hold water for various urban and agricultural uses including flood control. Following the guidelines below, model the (ever-decreasing) volume of water in our local reservoirs during Austin's drought this summer.

  During a drought, the only water entering a reservoir is due to feeder streams and rivers. Assume that the sum of all such inputs is 200 cubic feet per second. Further assume that the sum of all outflows is initially 250 cubic feet per second, but increases continuously by one cubic foot per second each day due to evaporation and increased water demand (before we started rationing!). Finally assume that the reservoir initially holds five billion cubic feet of water. After converting all time units to days, write a differential equation describing the flow of water into and out of this lake. Solve this differential equation and plot the solution. Assuming the parameters remain constant right up until the last drop of water, how many days will it take for the reservoir to empty to half of its initial level? How long will it take until 1 cubic foot of water remains?

  What would change if Austin rationed water usage? How much would they need to curtail water usage to prevent the reservoir from falling below half-full?

- 3. Carbon cycle

  One class of environmental models tracks the flow of some element or compound through an ecosystem. This project examines the flow of carbon through litter on a forest floor.

  In this context, litter is naturally-occurring debris such as leaves, branches and deadfalls (not discarded water bottles). A boundary for the system is set up, and only litter within this region is considered. Thus carbon is measured in density units: grams of carbon per square meter ($g\ C/m^2$).

  Here are the modeling assumptions: 1) Carbon coninuously enters the system through litterfall at a constant rate, z. 2) Carbon continuously leaves the system through two avenues – carbon dioxide produced in respiration, and the conversion of litter into humus (called humification). Even though the carbon in humus has not left the physical system boundary, it is no longer in the litter, just as carbon in the trees before the leaves fell was in the physical boundary, but not yet in the litter. 3) The rate of litterfall is constant; the rate of carbon removal from both avenues is proportional to the amount of carbon present. 4) Initially there is no carbon in the litter. This approximates the situation after a ground fire (a forest fire which burns the underbrush, but does not kill the trees).

  Set up the differential equation for this model, and solve it symbolically, with your favorite initial values.

  For a temperate forest, it is reasonable to assume a rate of litterfall of 240 $g\ C/m^2/yr$ and a proportionality constant for carbon removal of $0.4/yr$. Using this information, graph a solution curve for 50 years.

Of course, these examples are open to modification to suit your interests. For example, if you are a biochemist more interested in enzyme kinetecs than pharmacokinetics, you could use change the MME model above to reflect catalysis.